



Semi-final (EN)

03.02.2026

Instructions:

- Write your full name and school on each sheet.
- Clearly indicate which sub-/question you are answering.
- Start a new question on a new sheet of paper.
- Explain your reasoning and indicate intermediary calculations.
- Number your pages.

Formulae

Kinematics (UAM)

$$x = \frac{1}{2}at^2 + v_0t + x_0$$

$$v = at + v_0$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

Forces

$$F = ma$$

$$F_f \leq \mu N$$

Work, Energy, Power

$$W = Fd \cos \theta$$

$$E_{cin} = \frac{1}{2}mv^2$$

$$E_{pes} = mgh$$

$$E_{el} = \frac{1}{2}kx^2$$

$$P = \frac{W}{t} = Fv$$

Momentum

$$p = mv$$

$$F = \frac{\Delta p}{\Delta t}$$

Thermal concepts

$$Q = mc\Delta\theta$$

$$Q = mL$$

Ideal gas laws

$$p = \frac{F}{A}$$

$$pV = nRT = Nk_B T$$

$$E_K = \frac{3}{2}k_B T$$

Oscillations and waves

$$T = \frac{1}{f}$$

$$c = f\lambda$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Electricity

$$I = \frac{Q}{t}$$

$$F = k \cdot \frac{|q_1q_2|}{r^2}$$

$$V = \frac{W}{q}$$

$$E = \frac{F}{q}$$

$$V = RI$$

$$P = VI = RI^2 = \frac{V^2}{R}$$

$$R = R_1 + R_2 + \dots + R_n$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

$$\rho = \frac{RA}{L}$$

Electro-magnetism

$$F = qvB \sin \theta$$

$$F = BIL \sin \theta$$

Circular motion

$$v = \omega r$$

$$a = \frac{v^2}{r}$$

Gravitation

$$F = G \frac{mM}{r^2}$$

$$g = \frac{F}{m}$$

Quantum physics

$$E = hf$$

$$\lambda = \frac{hc}{E}$$

Optics

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$$

Question 1: Mechanics (20 points)

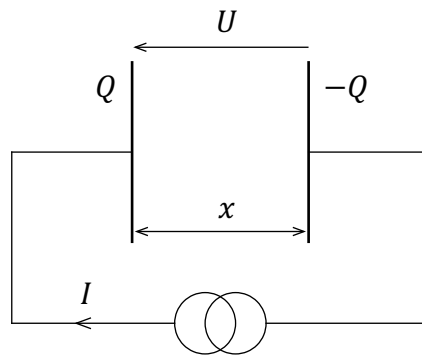
A firework rocket is launched vertically upwards and explodes at a height y_0 at the peak of its trajectory. It propels burning fragments in all directions (i.e. the fragments have different initial angles α_0 , where α_0 is the angle between the initial velocity vector \vec{v}_0 and the horizontal direction), but all with the same speed v_0 . The solidified metal particles fall to the ground without air resistance.

- 1) Give the equation of the height $y(t)$ of a fragment as a function of y_0 , α_0 and v_0 . **(2)**
- 2) Establish the expression for the time t_I at which a fragment reaches the ground as a function of y_0 , α_0 and v_0 . **(5)**
- 3) Establish the expressions for the velocity components $v_{x,I}$ and $v_{y,I}$ at impact as functions of y_0 , α_0 and v_0 . **(7)**
- 4) Establish the expression for the angle α_I the final velocity of a fragment makes with the horizontal at the moment of impact on the ground as a function of y_0 , α_0 and v_0 . **(3)**
- 5) For what initial angle α_0 is the angle α_I minimal?
From this, deduce the expression for the smallest possible angle of impact α_I . **(3)**

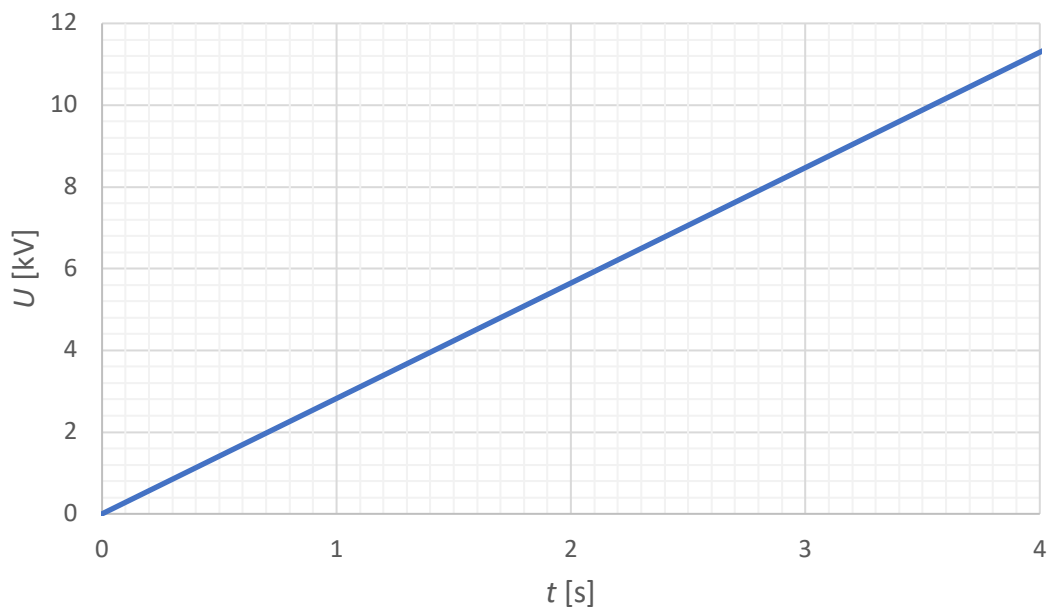
Question 2: Capacitor (20 points)

Consider a capacitor consisting of two flat, parallel plates with surface areas $S = 0,16 \text{ m}^2$, separated by a distance x , which carry opposite electric charges Q and $-Q$. The space between the plates is empty.

- 1) The capacitor, initially discharged, is charged using a high-voltage generator that delivers a constant electric current $I = 4 \mu\text{A}$ under a variable voltage.



The graph below shows the variation in voltage U across the capacitor as a function of time t .

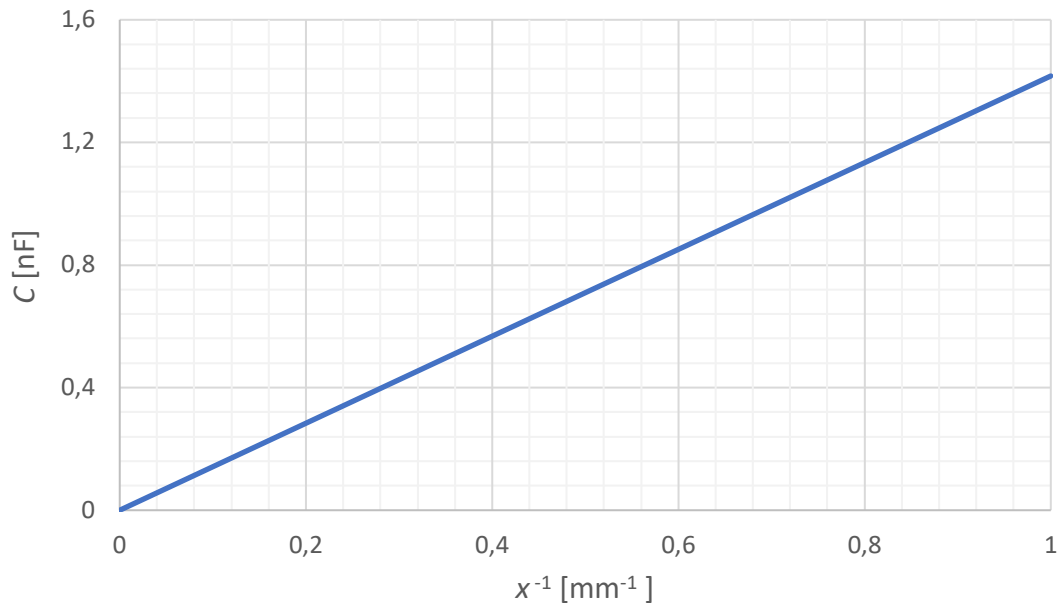


- a) Show that the charge Q of the capacitor is proportional to the voltage U . **(2)**
- b) The coefficient of proportionality is the *capacitance* C of the capacitor, defined by:

$$Q = CU$$

The SI unit of capacitance is the *farad* (F). Use the graph to determine the capacitance of the capacitor. **(3)**

- 2) The graph below shows the variation in capacitance C of the capacitor as a function of the inverse of the distance x between the plates.



Knowing that, for a given x , the capacitance of the capacitor is proportional to S , show that it can be written as:

$$C = \epsilon_0 \frac{S}{x}$$

Use the graph to determine the coefficient of proportionality ϵ_0 , which is a fundamental constant called the *permittivity of a vacuum*. **(5)**

- 3) Establish the expression for the electrical potential energy E_p of the charged capacitor, choosing $E_p = 0$ for an uncharged capacitor. Show that it can be written as: **(5)**

$$E_p = \frac{Q^2}{2C}$$

- 4) The capacitor is disconnected and isolated so that the charge Q remains constant. Use the results from points 2b) and 3) to show that the two plates attract each other with a force per unit area:

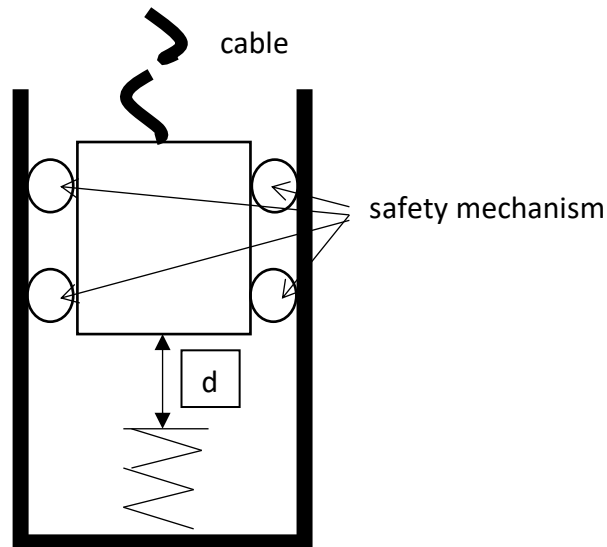
$$\frac{F}{S} = \frac{\sigma^2}{2\epsilon_0}$$

where $\sigma = \frac{Q}{S}$ is the capacitor charge per unit area.

Hint: Use $\Delta E_p = -W(\vec{F})$ when x changes. **(5)**

Question 3: Lift (20 points)

The cable of an elevator with mass m breaks when the elevator is stopped on the ground floor. The base of the elevator is then at a distance d above a spring with a spring constant k . A safety mechanism is triggered, creating a friction force F .



- 1) Indicate all forces acting on the lift during the different phases of motion and draw a diagram for each phase, annotating it with the relevant values. **(6)**
- 2) Calculate the maximum compression distance Δx of the spring by literally applying the principle of conservation of energy. **(8)**

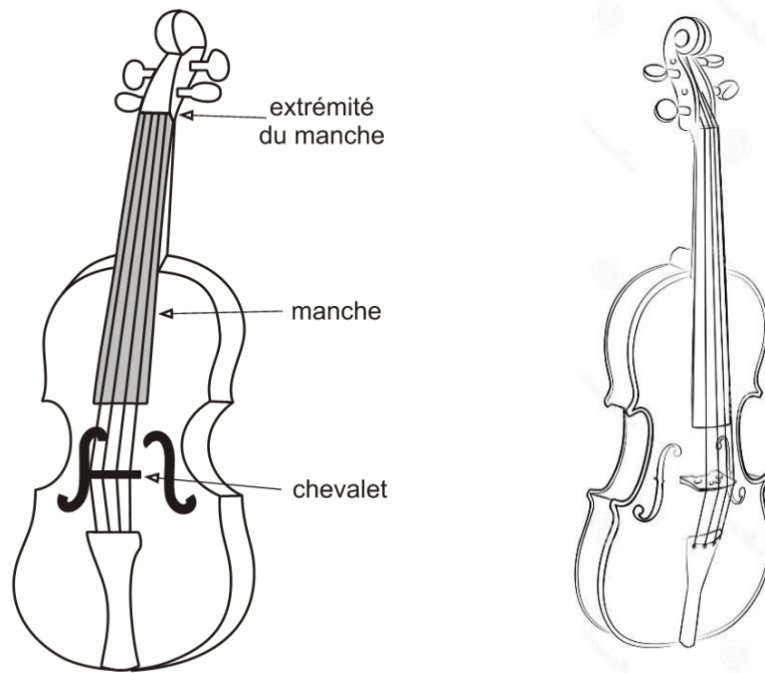
Numerical application:

$$m = 2.00 \cdot 10^3 \text{ kg} \quad d = 4.00 \text{ m} \quad k = 1.40 \cdot 10^5 \text{ Nm}^{-1} \quad F = 5.00 \cdot 10^3 \text{ N}$$

- 3) To what height h will the lift rebound? **(6)**

Question 4: Standing waves in a violin string (20 points)

When a violinist rubs a string with his bow, it vibrates in its fundamental state. In order to make it vibrate in a state with several antinodes, he uses the technique of *harmonics in flageolet*: he gently places his finger on the part of the string where he wants to create a vibration node without pressing the string against the neck. In this way, he creates standing waves along the entire length of the string (from the bridge to the end of the neck). If, on the other hand, he presses the string firmly against the neck, he simply shortens the vibrating length of the string.



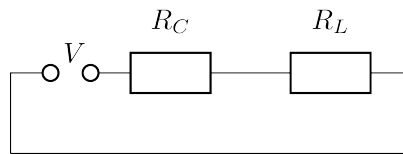
(*manche/Hals/neck ; chevalet/Steg/bridge; extrémité/Ende/end*)

- 1) The played string vibrates in its fundamental state.
 - a) Describe the motion of the points of one of the strings when it vibrates in its fundamental state. Add a sketch of the string at a moment when the points of the string have maximum/minimum elongation. **(3)**
 - b) The ratio of the fundamental frequencies of a string and its neighbouring string with a higher pitch is $2/3$. How do you explain that these frequencies are so different? (Assume that the tensions of the four strings are roughly the same.) **(3)**
 - c) Explain why the pitch becomes higher when the string is shortened. **(3)**

- 2) The violinist produces harmonics in flageolet on the A-string with a fundamental frequency of 440 Hz.
- a) The violinist places his finger so that the string vibrates with three antinodes. Draw a sketch of the string at a moment when the points on the string have maximum/minimum elongation. What is the frequency of vibration of the string? **(4)**
 - b) Where should he place his finger so that the string vibrates with 2 antinodes, 3 antinodes, 4 antinodes? **(3)**
 - c) To play the very high-pitched sound with a frequency of 3080 Hz, there are several possibilities for placing the finger. Give at least two. **(4)**

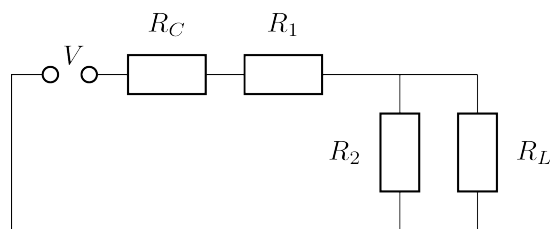
Question 5: Impedance matching (20 points)

We consider a voltage source with voltage V which is connected to a load with resistance R_L by a cable with resistance R_C .



- 1) Calculate the total resistance R_{tot} of the circuit and the power P_L delivered to the load resistance. **(4)**
- 2) Assume that R_C is fixed but R_L can be varied. Show that P_L is highest for $R_L = R_C$ and calculate P_L for this case. **(6)**

For $R_L = R_C$, the circuit is called “impedance matched” and the power output to the load is at a maximum. Because we cannot change R_C , this condition cannot be met for arbitrary load resistances R_L . To maximize power output for other loads, one can add additional resistors R_1 and R_2 , as shown in the figure:



- 3) Calculate the total resistance $R_{\text{tot},2}$ of the new circuit and the new effective load resistance R'_L . **(4)**
- 4) Show that the circuit is impedance matched if we choose $R_2 = R_L\sqrt{1 - R_L/R_C}$ and $R_1 = R_C\sqrt{1 - R_L/R_C}$. **(6)**