PHYSIKSOLYMPIAD LËTZEBUERG 2024

Semi-final (EN)

20.02.2024

Instructions:

- Write your full name and school on each sheet.
- Clearly indicate which sub-/question you are answering.
- Explain your reasoning and indicate intermediary calculations.
- Number your pages.

Formulae

Kinematics (UAM)
$x = \frac{1}{2}at^2 + v_0 t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces
$F = ma$

$F_f \leq \mu N$

Work, Energy, Power
$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum
$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts
$Q = mc\Delta\theta$

$Q = mL$

Ideal gas laws
$p = \frac{F}{A}$

$pV = nRT = Nk_BT$

$E_K = \frac{3}{2}k_BT$

Oscillations and waves
$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricity
$I = \frac{Q}{t}$

$F = k \cdot \frac{|q_1 q_2|}{r^2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism
$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion
$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation
$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics
$E = hf$

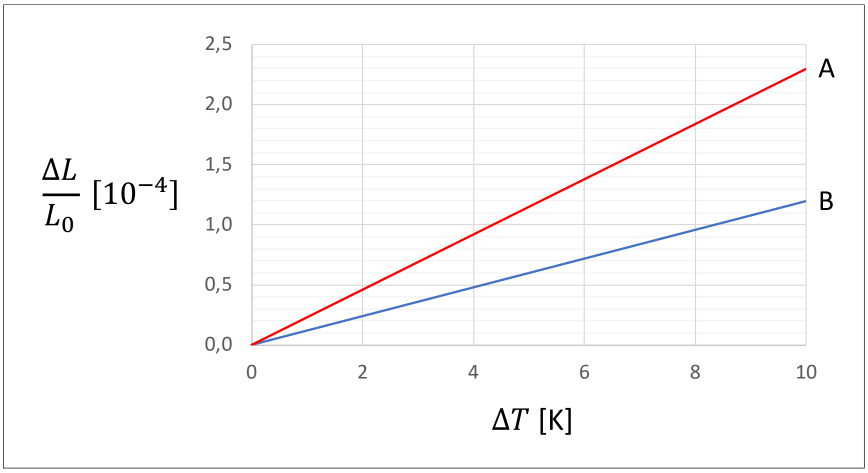
$\lambda = \frac{hc}{E}$

Optics
$n_1\sin\alpha_1 = n_2\sin\alpha_2$

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$$

Question 1: Bimetallic strip (20 marks)

1) A metal blade has a length L_0 at temperature T_0. When the temperature of the blade increases by ΔT its length increases to the length $L = L_0 + \Delta L$. The graph below shows the relative variation in length as a function of temperature variation for two different metals A and B.

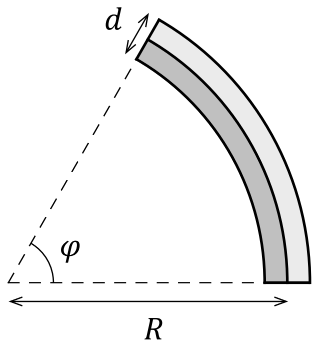


From the graph, derive the general expression for the length L as a function of L_0 and ΔT and determine the numerical values of any parameters for the two metals. (4)

2) A bimetallic strip consists of a metal blade A and a second metal blade B of the same length L_0 and the same thickness $d/2$. The two strips are glued together lengthwise so that the bimetallic strip is straight at temperature T_0. It deforms when its temperature varies, the variation in its thickness can be neglected.

Establish an expression for the radius of curvature R of the bimetallic strip, as defined in the figure below, as a function of the increase in its temperature ΔT and its thickness d.

Hint: consider the angle at the centre φ.

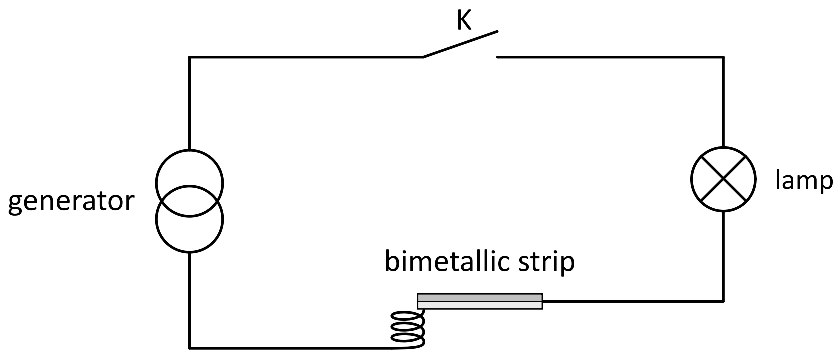


Show that for temperature variations of less than $100\ \mathrm{K}$ the radius of curvature is given approximately by (in SI units):

$$R = 4{,}5 \cdot 10^4 \frac{d}{\Delta T}.$$

(7)

3) The bimetallic strip is connected in series with a lamp and a switch K to a current generator.



The lamp has the characteristics ($6\ \mathrm{V}$; $12\ \mathrm{W}$). The thickness of the bimetallic strip is $d = 0{,}2\ \mathrm{mm}$, its resistance is $0{,}8\ \Omega$ and its heat capacity is $1{,}6\ \mathrm{J/K}$. The generator supplies a constant current so that the voltage across the lamp is $6\ \mathrm{V}$.

When switch K is closed, the temperature of the bimetallic strip is T_0 so it is straight, the circuit is closed. The deformation of the bimetallic strip due to the increase in its temperature will open the circuit when the radius of curvature reaches $45\ \mathrm{cm}$.

In a first approach, the change in resistance of the bimetallic strip with temperature and the transfer of heat to the surrounding air are neglected.

After what time Δt will the lamp go out? (5)

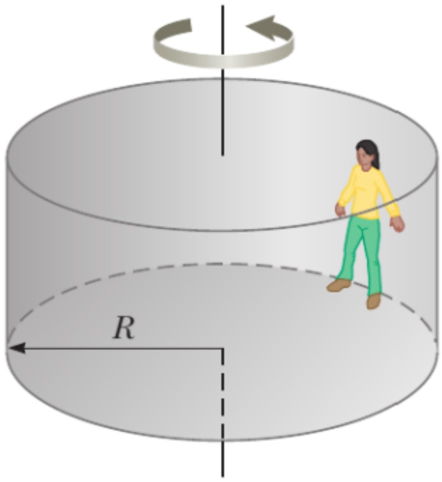
4) Under the same conditions, how does the value of Δt calculated change by considering that

   a) the resistance of the bimetallic strip increases with temperature? (2)

   b) heat transfer to the surrounding air is not negligible? (2)

Question 2: Mechanical problem (20 marks)

The Rotor is an amusement park game, first shown at the "Oktoberfest" in 1949. This year, it was also present at the "Schueberfouer" in Luxembourg city.



It consists of a cylinder of radius R which rotates around its axis at an angular velocity ω so that any person of mass m inside remains "stuck" to the wall when the floor is lowered.

The coefficient of static friction between the person and the wall is given by μ. The maximum value for the coefficient of friction is 1. The angular velocity varies between 0 and 30 revolutions per minute, $R = 4 \text{ m}$; $m = 60 \text{ kg}$.

1) Draw a diagram showing all the forces exerted on the person, relative to an inertial frame of reference, when the cylinder rotates at a low angular speed, with the floor still in contact with the feet. For each of these forces, specify the body exerting the force. On the same diagram, show the person's acceleration vector. (4)
2) Answer the same questions than in 1) if the cylinder rotates fast enough that the floor can be lowered without the person sliding down. (3)
3) Under the conditions of 2) express the intensities of the forces as a function of m, ω, R and μ. Calculate μ knowing that the Rotor rotates with maximum speed. (4)
4) Calculate, again for maximum speed,
   a) the acceleration of the person; express it as a function of the acceleration of gravity g; (2)
   b) the resultant force exerted on the person; express it as a function of the person's weight. (2)
5) At the end of the run, the angular velocity gradually decreases. How does the coefficient of friction μ change if the person remains "stuck" to the wall? Explain. Calculate the minimum angular velocity (in revolutions per minute) at which the person remains "stuck" to the wall. (4)
6) Explain why, when the angular velocity decreases, some people slide downwards while others remain perfectly "stuck" to the wall. (1)

Question 3: Mechanical oscillations (20 marks)

We consider a mass m which is connected to a spring with spring constant D such that it can undergo vertical oscillations. When the mass is at coordinate $y = 0$, the spring is relaxed and exerts no force. When the mass is displaced up or down, the restoring force of the spring is $F = -Dy$. Hence, the equation of motion $F = ma$ leads to

$$m\ddot{y}(t) = -Dy(t)$$

Let us assume that we can neglect the gravitational force. The solution of this differential equation is $y(t) = A\cos(\omega t) + B\sin(\omega t)$.

1) We assume that the mass is released at time $t = 0$ at a height $h > 0$ with a vertical velocity $v(0) = 0$. Use these initial conditions and the equation of motion to determine ω, A and B. (4)
2) Calculate the time t_0 when the mass reaches the point $y = 0$ and the time t_1 when it reaches its lowest point. (6)

From now on, we assume that the region at $y < 0$ is filled with water. Once the mass enters the water, it experiences a friction force $F_f = -\gamma v$, which is oriented opposite to the velocity of the mass. The friction force causes the mass to lose energy.

3) Calculate the lost energy W from the time where the mass enters the water to the time where it changes direction. You can assume that the friction is very weak, such that the velocity $v(t)$ in F_f is the velocity of the mass without friction. (4)

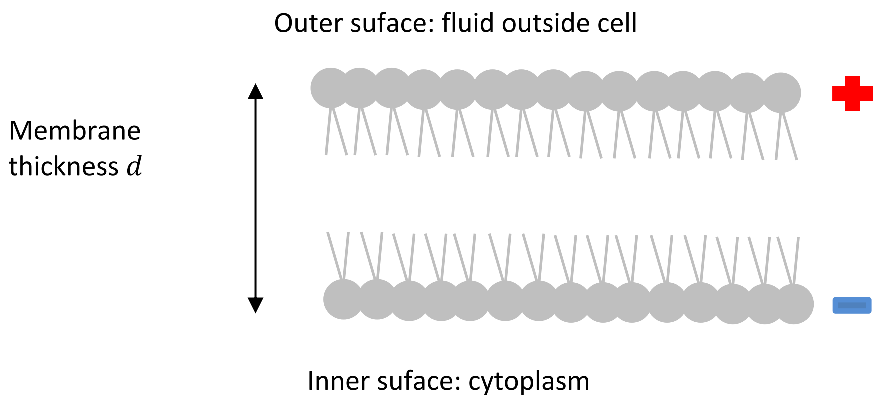
   Hint: Use $W = -\int_{t_0}^{t_1} F_f\,(t)\,v(t)\,dt$ to calculate the lost energy. You may use the integral

$$\int dx\,\sin^2(x) = \frac{x}{2} - \frac{1}{4}\sin(2x)$$

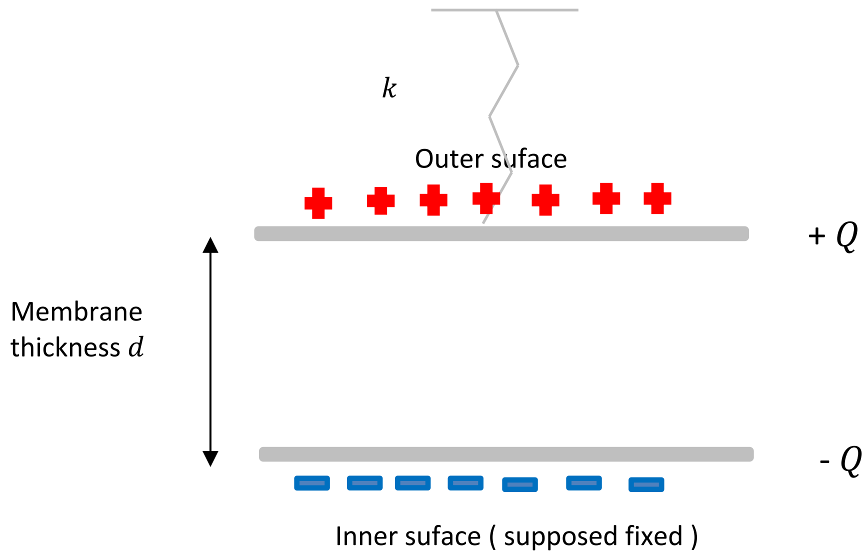
4) The result of the previous calculation is $W = \pi h^2 \gamma\omega/4$. Use this to calculate the new maximum height that the mass will reach after leaving the water, again assuming that the friction force and the lost energy are small. (6)

Question 4: Modelling a neuron membrane (20 marks)

The wall of a neuron is made from an elastic membrane, which resists compression in the same way as a spring. It has an effective spring constant k and an equilibrium thickness d_0. Assume that the membrane has an infinitely large area A.



It can be assimilated to an elastic capacitor with a spring constant k, an area A and a separating distance d between the plates.



The neuron has "ion pumps" that can move K^+ and Na^+ ions across the membrane. In the resulting charged state, positive and negative ionic charge is arranged uniformly along the outer and inner surfaces of the membrane, respectively. The permittivity of the membrane is ε.

Read the following information carefully before you start:

- The electric field of a flat large charge layer (one plate) is given by $E = \frac{\sigma}{2\varepsilon}$ where σ is the charge density $\sigma = \frac{Q}{A}$ unit: Cm^{-2}.
- Derivatives:

  power rule $f(x) = a\,x^n$ derivative $\frac{d}{dx}f(x) = n\,a\,x^{n-1}$

derivative of a sum is equal to sum of derivatives.

1) Model of a capacitor:

   Draw a diagram of the two parallel horizontal charged plates.
   Add the electric field vector of the field created by each plate above, between and below the plates.

   Deduce from your diagram that the electric field intensity

   - between the plates is given by $E = \frac{Q}{\varepsilon A}$ and
   - on either side $E = 0$.

   Explain. (5)

2) Model of the neuron:

   Suppose that, after some amount of work is done by the ion pumps, the charges on the outer and inner surfaces are + Q and – Q as shown in the diagram.

   a) What would happen to the two layers/surfaces if free to move, i.e. no spring / no elasticity present? Explain. (2)

   b) i. What is the electrical force exerted by the inner charged surface on the outer charge Q? (2)

      With the spring present, the spring force balances the electric force between the two sides of the membrane.

      ii. Show that the thickness d of the membrane is given by $d = d_0 - \frac{Q^2}{2\varepsilon A k}$. (4)

3) Derive an expression for the voltage V between the outer and inner surfaces of the membrane in terms of Q and the other parameters ε, A, d using your answers from 1) and 2).

   Explain qualitatively how the voltage V changes with Q and deduce that there must be a maximum charge and voltage beyond which the membrane will collapse. (7)

4) *Bonus*: Derive the expression for the voltage with respect to Q and determine the maximum charge Q allowed before the membrane collapses. Calculate the corresponding maximum voltage. (4)