PHYSIKSOLYMPIAD LËTZEBUERG 2018

Semi-final(EN)

6.2.2018

Formulas

Kinematics (UAM)

$x = \frac{1}{2}at^2 + v_0 t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces

$F = ma$

$F_f \leq \mu N$

Work, Energy, Power

$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts

$Q = mc\Delta\theta$

$Q = mL$

Ideal gas

$p = \frac{F}{A}$

$pV = nRT = Nk_B T$

$E_K = \frac{3}{2}k_B T$

Oscillations and waves

$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricty

$I = \frac{Q}{t}$

$F = k \cdot \frac{q_1 q_2}{r_2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism

$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion

$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation

$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics

$E = hf$

$\lambda = \frac{hc}{E}$

Optics

$n_1\sin\alpha_1 = n_2\sin\alpha_2$

$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$

Problem 1: Galaxies and Gravitation (20 points = 6+5+4+5)

A galaxy is a local agglomeration of stars which are not uniformly distributed. The stars form a galactic disk whose height is very small compared to its radius. The galaxy that hosts the Sun with the Earth is called the Milky Way and is an example of a spiral galaxy.

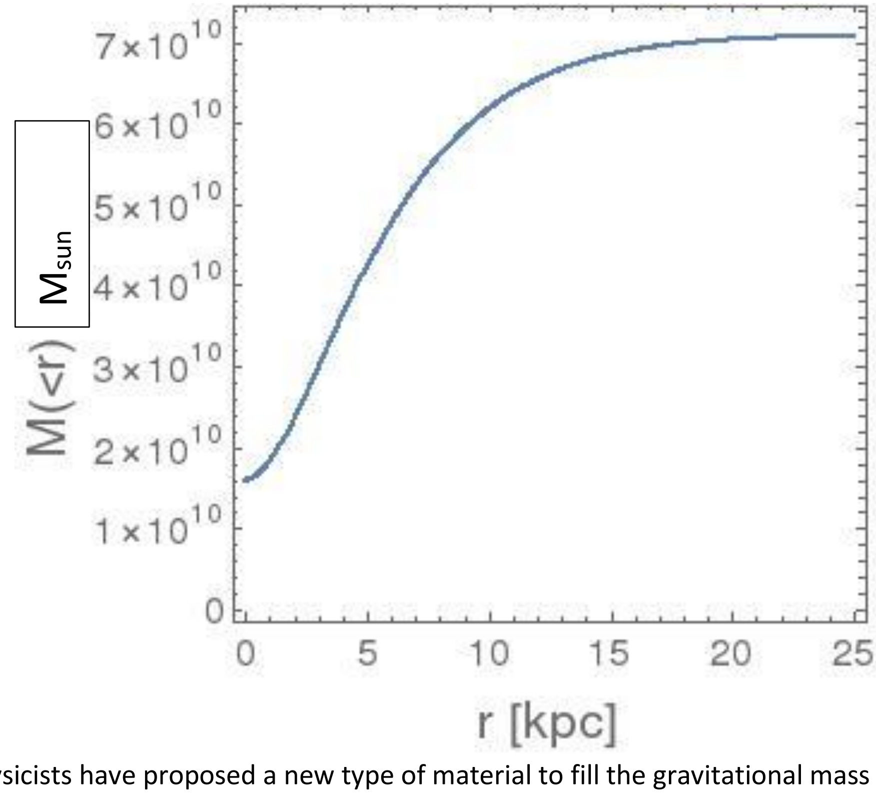
a) Give the expression of the rotation speed v_E of a star of mass m_E which is at a distance R from the galactic centre. Suppose that only the stars inside the sphere of radius R, of mass M (R), contribute to the gravitational force acting on the star in question.

b) Knowing that the star of our solar system, the Sun, has a rotation speed of 230 kms^{-1} and that it is at a distance of 8000 pc from the galactic centre, calculate the mass of material contained in the inside this radius.

   Info: The parsec (pc) is a unit of astronomical length. $1 \text{ pc} = 3.08567758 \times 10^{16}$ m.

c) The following graph shows the cumulative mass of the stars of the Milky Way as a function of the distance to the galactic centre. What is the mass of stars inside the 8 kpc sphere? Work out if all the mass involved in the gravitational attraction lies in the stars.

   The mass of the sun is $M_{sun} = 2.0 \times 10^{30}$ kg.

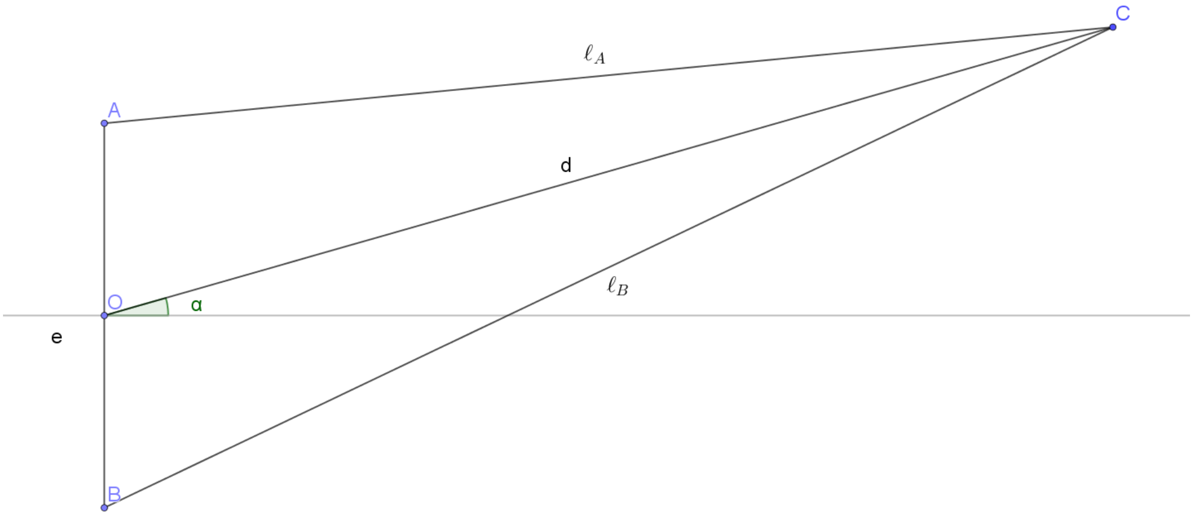


d) Astrophysicists have proposed a new type of material to fill the gravitational mass deficit observed in galaxies. Since this matter does not interact with light (and is therefore "invisible"), they call it "dark matter".

   What is the mass of dark matter (in M_{sun}) within 25 kpc, knowing that (in this particular model that was used to make the graph) the proportion of the mass of dark matter in relation to the total mass is 76.9 %?

Problem 2: Waves / oscillations (20 points= 5+4+1+6+2+2)

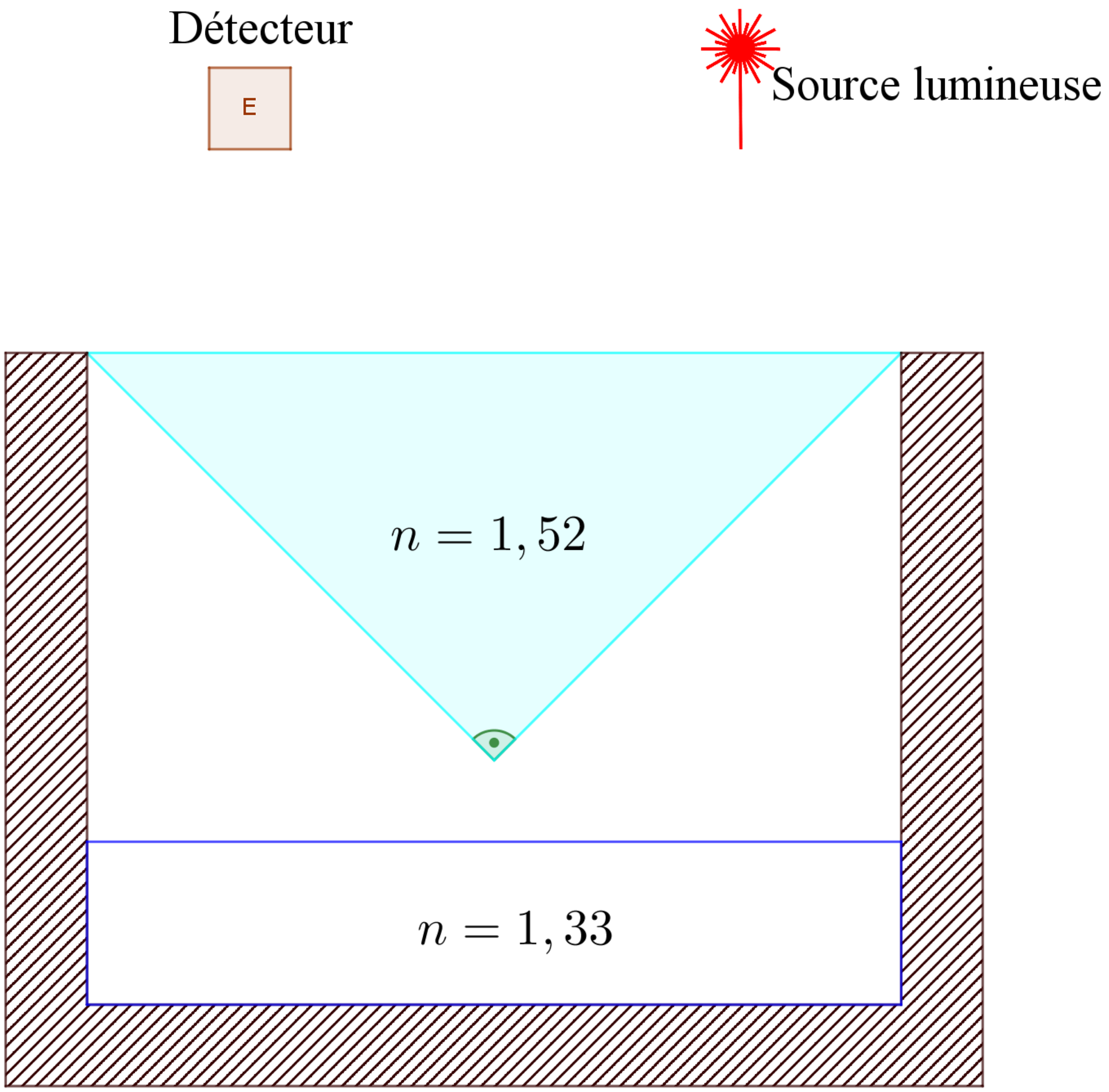
Two radio antennas A and B, spaced by a distance e, emit a sinusoidal electromagnetic signal at a fixed frequency f. The speed of propagation of the waves is the speed of the light c and one calls λ the wavelength. Antennas can be considered to act as coherent point sources. The difference in the path travelled by the two waves emitted by A and B respectively to arrive at a given point C is called the difference in path $\Delta\ell = \ell_A - \ell_B$. If the difference in path corresponds to an integer multiple of the wavelength $(\Delta\ell = n \cdot \lambda \quad n\ is\ an\ integer)$, the waves interfere constructively, and the signal is reinforced. If the path difference is an odd multiple of half a wavelength $\left(\Delta\ell = (2n+1) \cdot \frac{\lambda}{2}\right)$, the waves interfere destructively, and the signal is cancelled.



1. A distant receiver $(d \gg e)$ picks up the signal from both antennas. At which angles α will reception be a maximum? (5p)
2. This technique is used to focus the power radiated by the antennas in a given direction increasing the efficiency. In order to optimize the issue, it is necessary to limit the number of emission maxima. Under what condition could we observe only one maximum? (4p)
3. Calculate the separation “e” in the limiting case of 1 maximum for f = 92.5 MHz. (1p)
4. For a system of two antennas, it is desired to vary the maximum transmission direction without having to turn the antennas mechanically. This is why the engineers introduce a phase shift φ of the signal into one of the antennas. This is equivalent to a spatial shift $(\Delta\ell_{phase})$ of the antenna where $\Delta\ell_{phase} = \frac{\varphi}{2\pi}\lambda$ in the respective transmission direction. Express the angle of the maximum reception direction α as a function of the other variables. (6p)
5. Calculate the phase shift required to deflect the maximum emission direction by 12 °. Where: e = 12 m and f = 107.7 MHz. (2p)
6. From a technical point of view, when constructing a directional antenna system, one often uses a row of antennas spaced from the same distance. What is the advantage of such a device? (2p)

Problem 3: Optics - Refraction (20 Points = 5 + 4 + 3 + 3 + 5)

In the device below a light source (source lumineuse) emits a beam of incident light on a right angle prism placed in a tank.



a) Draw the exact path of the light beam when the prism is surrounded entirely by air.

b) Draw the exact path of the beam when the tank is filled with water (i.e. surrounding the bottom surface of the prism). The refractive index of water is 1.33. Explain your answer.

c) E is a photocell that produces an electrical voltage when illuminated. Explain briefly how this device works and what it could serve.

d) What is the minimum value for the refractive index of a liquid in the tank for the device to still work?

e) What is the maximum value for the refractive index of the prism so that the device is still working with water? What are the prisms that can be used for this device from the list of prisms listed below?

Glass	**Refractive index**
Pyrex	1,474
Germanium glass	1,608
Schott IR25	2,728
Diamond	2,418
Crown FK5	1,487
Flint LaFN2	1,744
Flint LaSF30	1,803

Problem 4: Helium at very low temperatures (16 points = 4+3+4+3)

We are interested here in the basic operation of a "1K pot", a tool that can achieve temperatures between 1 K and 5 K, used in the first stage of cooling in very low temperature systems. This tool is normally in thermal contact with the sample which one wishes to cool, but in this problem, one will treat only the case of an isolated 1K pot.

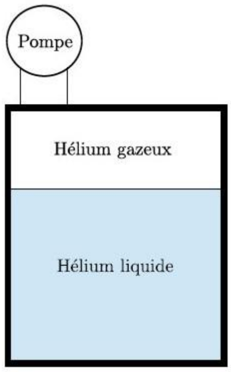


Fig. 1: Normal

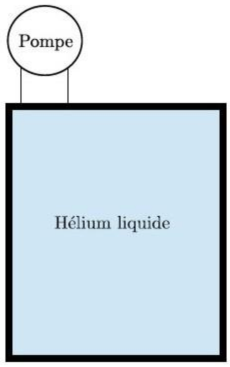


Fig. 2: Full

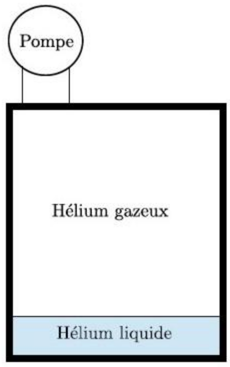


Fig. 3: Empty

Fig 1. shows a bath of liquid helium isolated from any hot body that could boil it, and is at ***saturation vapor pressure***. The top of the bath is connected to a vacuum pump that is powerful enough to draw helium gas over the bath, but not liquid helium. When the pump is running, it can be considered that there is a vacuum above the liquid helium.

saturation vapor pressure is the pressure at which the transitions between the liquid (or solid) and gaseous phases are at equilibrium. If the pressure of a gas is higher than the saturation vapor pressure, there will be condensation until equilibrium of the pressure and the saturation vapor pressure. You can assume that this saturation vapor pressure varies very little with temperature.

1. (4 pts) Explain why helium starts to evaporate when you start the pump.
   What happens if you stop the pump?

2. (3 pts) Consider 300 mL of liquid helium at 4.3 K and at atmospheric pressure; how much heat energy does it take to evaporate this liquid?

3. (4 pts) Explain why the temperature of the liquid layer very close to the surface drops when the pump is running.
   Which two factors have the most influence on the cooling rate of the bath? Explain qualitatively.

4. (3 pts) Now consider the cooling rate of liquid helium in Figures 2 and 3? Compare it to that of Fig. 1. Explain qualitatively.

5. (2 pts) We now want to use this principle to make a refrigerator. What will prevent the refrigerator from maintaining a stable temperature over time in the model shown in FIG.1? What should be done to fix it?

   Bonus (+1 pt): What are the technical parameters that allow the temperature to be adjusted?

Physical constants:

Atmospheric vaporization temperature of helium: $T_v = 4{,}3K$

Latent heat of vaporization of liquid helium at saturated vapor pressure: $L_v = 81\ J/mol$

Molar mass of helium: $M_{He} = 4{,}0026\ g/mol$

Density of liquid helium as a function of temperature:

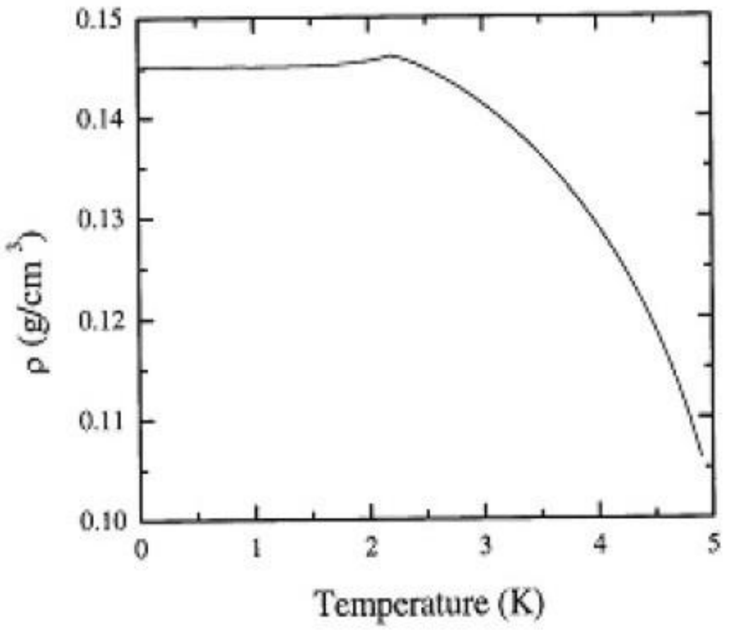
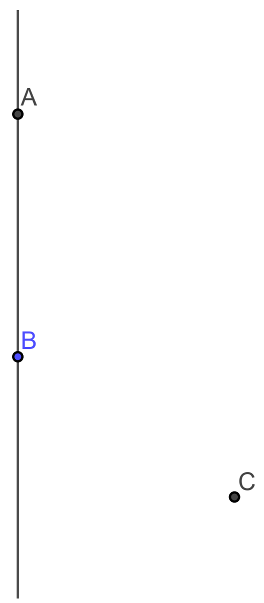


Fig. 4: Source: "The Observed Properties of Liquid Helium at the Saturated Vapor Pressure" by Russell J. Donnelly and Carlo F. Barenghi

Problem 5: Optics - speed of light (8 + 2 = 10)

A particle of cosmic radiation moves at a speed close to the speed of light. It enters a large container filled with a luminescent liquid of refractive index n equal to 1.6. It moves on a line AB as shown in the figure. A detector is placed at point C. It simultaneously records two light flashes from points A and B.



1. Using the scale diagram above, determine the velocity of the particle. Express the result in scientific notation and estimate the number of significant digits.

2. Determine the position of other points of the liquid where the flashes could be seen simultaneously.