PHYSIKSOLYMPIAD
LËTZEBUERG 2022

# Semi-finals (EN)

**28.02.2022**

## Instructions

- Indicate your full name and the name of your school on each page.
- State clearly on each page to which question your solution relates.
- Explain your approach and present your interim calculations
- Number each page.

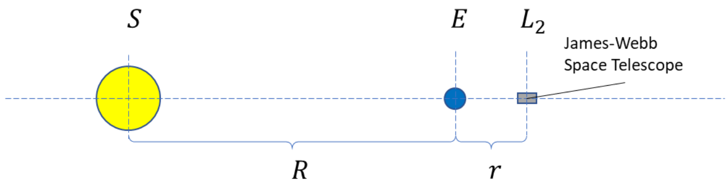
## Formulae collection

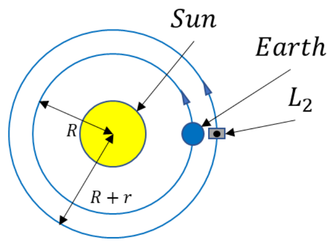
| cinematics | forces | circular movement |
|---|---|---|
| $x(t) = \frac{1}{2}at^2 + v_o t + x_0$ <br> $v(t) = at + v_0$ <br> $\Delta(v^2) = 2a \cdot \Delta x$ | $\vec{F} = m \cdot \vec{a}$ <br> $F_f \leq \mu \cdot F_N$ | $v = \omega \cdot r$ <br> $a = \frac{v^2}{r}$ |
| **momentum** | **calorimetry** | **ideal gas** |
| $\vec{p} = m \cdot \vec{v}$ <br> $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ | $Q = m \cdot c \cdot \Delta\theta$ <br> $Q = m \cdot L$ | $p = \frac{F}{A}$ <br> $pV = nRT$ <br> $pV = Nk_B T$ <br> $E_K = \frac{3}{2} \cdot k_B \cdot T$ |
| **work/power/energy** | **electricity** | **oscillations and waves** |
| $W = F \cdot d \cdot \cos(\theta)$ <br> $E_c = \frac{1}{2}mv^2$ <br> $E_p = mgh$ <br> $E_{el} = \frac{1}{2}kx^2$ <br> $P = \frac{W}{t}$ <br> $P = F \cdot v$ | $I = \frac{Q}{t}$ <br> $F = \frac{1}{4\pi\epsilon_0}\frac{\lvert q_1\rvert \cdot \lvert q_2\rvert}{r^2}$ <br> $U = \frac{W}{q}$ <br> $E = \frac{F}{q}$ <br> $U = R \cdot I$ <br> $P = U \cdot I = R \cdot I^2 = \frac{U^2}{R}$ <br> $R = \sum R_i$ <br> $\frac{1}{R} = \sum \frac{1}{R_i}$ <br> $\rho = \frac{R \cdot A}{L}$ | $T = \frac{1}{f}$ <br> $c = \lambda \cdot f$ <br> $T = 2\pi \cdot \sqrt{\frac{l}{g}}$ <br> $T = 2\pi \cdot \sqrt{\frac{m}{k}}$ |
| **electromagnetism** | **gravitation** | **optics** |
| $F = q \cdot v \cdot B \cdot \sin(\theta)$ <br> $F = B \cdot I \cdot L \cdot \sin(\theta)$ | $F = \frac{G \cdot m_1 \cdot m_2}{r^2}$ <br> $F = m \cdot g$ | $n_1 \sin(\alpha) = n_2 \sin(\beta)$ <br> $f^{-1} = g^{-1} + b^{-1}$ |

| quantum mechanics |
|---|
| $E = h \cdot f$ <br> $\lambda = \frac{h \cdot c}{E}$ |

# Task 1 – James-Webb Space Telescope [3 + 5 + 7 = 15 P]

On 25 December 2021, the mission of the James-Webb Space Telescope was launched. James-Webb is a space telescope that can look into deep space using infrared sensor technology. To avoid electromagnetic interference from radiation of Sun, Earth and Moon, James-Webb will be placed at a large distance relative to Earth, the so called Lagrange point $L2$. A Lagrange point is a position in a system of two celestial bodies (here the Sun and the Earth) at which bodies with a tiny mass (e.g. asteroid, space probes, etc.) can orbit a body with a higher mass (here the Sun) without any propulsion, but not the body with the lower mass (here the Earth). A body at $L2$ therefore also has the same angular velocity $\omega$ as the Earth. The aim of this task is to determine the distance $r$ between Earth and $L2$. To do this, answer the following intermediate steps.




1) Derive the angular velocity $\omega_E$ of the Earth as a function of the gravitational constant $G$, the solar mass $M_S$ and the orbital radius $R$.

2) The mission aims to positionthe telescope at the Lagrange point $L2$. The telescope has no influence on the Earth-Sun system because of its low mass. At $L2$, the telescope is held in a circular orbit by the gravitational pull of the Sun and Earth. Set up an equation that describes the unknown orbital radius $r$ without solving this equation completely. The equation should only contain the Sun's mass $M_S$, the Earth's mass $M_E$, the orbital radius $R$ and the gravitational constant $G$.

3) Simplify the previous established equation with the following approximation: Since the radius is $r \ll R$ ; $\frac{r}{R} \ll 1$, one can write approximately:

$$\frac{1}{\left(1+\frac{r}{R}\right)^2} \approx 1 - \frac{2r}{R}$$

Determine the distance $r$ between the earth and the telescope.

Constants:

- mass of the sun: $M_S = 1{,}99 \cdot 10^{30}$ kg
- mass of the earth: $M_E = 5{,}97 \cdot 10^{24}$ kg
- distance sun – earth: $R = 1{,}47 \cdot 10^{11}$ m

## Task 2 – Ice pellets and Rain [2 + 1 + 3 + 9 + 3 = 18 P]

Water droplets or ice pellets form at a height of a few kilometres in the atmosphere and fall through the atmosphere towards the ground.

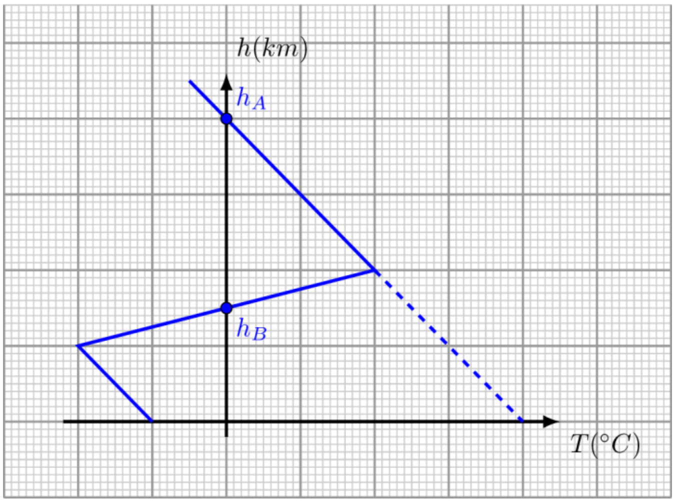
1) Calculate the speed of an ice pellet dropping from a height of 4km assuming there is no air resistance.

2) Explain why this result is unreasonable.

3) Assume that half of the potential energy of an ice pellet at 0°C is converted into internal energy of the pellet during the descent. What would the state of the pellet or droplet be when it hits the ground?

| For the remainder of the exercise, we will neglect these viscous losses |
|---|

While falling through the air, an ice pellet encounters different surrounding temperatures shown below in the temperature profile below:




Assume that while travelling from A to B, the ice pellet completely melts and that heat conduction for ice and water is perfect.

4) Determine the ratio $\eta$ of frozen and total mass of the droplet when reaching the ground. Explain your reasoning and specify any additional assumptions you made.

In a different scenario, the temperature follows the dashed line.

5) Determine the altitude $h_C$ for which the pellet would melt completely considering the new temperature profile.

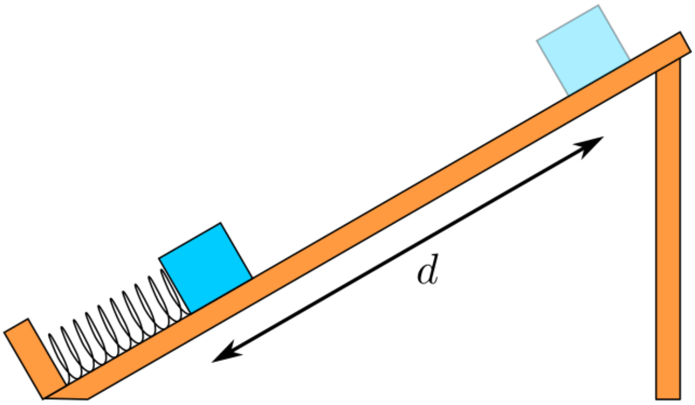
**Tip:** the rate of heat exchange for small temperature gradients follows the following law:

$$\frac{dQ}{dt} = \kappa \Delta T = \kappa (T_{in} - T_{out})$$

- Chain rule for derivation: $\frac{dT}{dt} = \frac{dT}{dx} \cdot \frac{dx}{dt}$
- specific heat of water: $c_{Wasser} = 4{,}2 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
- specific heat of ice: $c_{Eis} = 2,1 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
- latent heat for the melting of ice: $L = 334 \text{ kJ} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$

## Task 3 – Inclined plane and spring [5 + 5 + 5 = 15 P]

A weightless spring with a spring constant $k = 800$ N/m and uncompressed length of 1 m is placed on a plane that is inclined at an angle of 45° to the horizontal on a frictionless slope.




Suppose now that the spring is compressed to a length of 0,5 m and a mass of 4 kg is placed at its free end before the spring is released.

1) If the mass is not attached to the spring, how far up the slope will it travel before coming to rest?

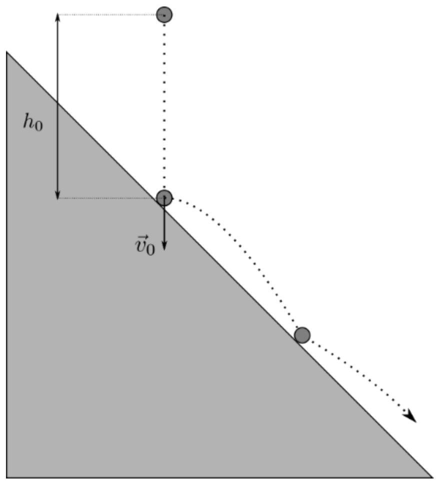
2) If the mass is attached to the spring, how far up the slope will it travel before coming to rest?

3) Suppose now that the inclined plane is not frictionless, but rather has a coefficient of friction μ. If we presume that the mass is attached to the spring and if the mass comes to rest just as the spring reaches its uncompressed length, what would be the coefficient of friction. Do you think the result is plausible?

**Hint:** $\mu = \frac{F_{fr}}{F_n}$ where $F_{fr}$ is the intensity of the frictional force and und $F_n$ the intensity of the normal pressure force

## Task 4 – Bounce, bounce, bounce [1 + 2 + 4 + 3 + 6 +4 = 20 P]

A ball is dropped vertically from a height $h_0$ and hits a plane inclined at 45° to the horizontal. At the point of impact, the velocity is given as $\vec{v}_0$. The ball undergoes a perfectly elastic collision and rebounds, hits the plane, rebounds again and so on down the inclined plane.




In the subsequent calculations the x-direction is chosen parallel to the surface of the plane and the y-axis is perpendicular (normal) to the plane.

1) Draw the $x$ and $y$ components of the velocity ( $\vec{v}_x$ and $\vec{v}_y$) and acceleration ($\vec{a}_x$ and $\vec{a}_y$). at the point of impact on an annotated diagram.

2) Show that the impact speed is given by $v_0 = \sqrt{2gh_0}$ and that the drop time is given by $t_O = \frac{v_0}{g}$.

3) Write the equations of motion for $x(t), y(t), v_x(t)$ and $v_y(t)$ for the time interval between the first and second impact.

4) Show that the velocity component $v_y$ perpendicular to the plane has a constant magnitude.

5) Show that the speed after the n^th bounce is equal to

$$v_n^2 = \frac{[(2n+1)^2 + 1]}{2} v_0^2 \; n = 0,1, \dots$$

   $n = 0$ corresponds to the first bounce at the first point of impact after the free fall of the ball.

6) Show that the angle $\theta_n$ between the velocity and the plane is equal to

$$\tan\theta_n = \frac{1}{2n+1}$$

   Deduce from your result that as the ball keeps on rebounding it gets closer and closer to the plane.