PHYSIKSOLYMPIAD LËTZEBUERG 2021

Semi-final (EN)

25.02.2021

Instructions :

- Write your full name and school on each sheet.
- Clearly indicate which sub-/question you are answering.
- Explain your reasoning and indicate intermediary calculations.
- Number your pages.

Formulae

Kinematics (UAM)

$x = \frac{1}{2}at^2 + v_0 t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces

$F = ma$

$F_f \leq \mu N$

Work, Energy, Power

$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts

$Q = mc\Delta\theta$

$Q = mL$

Ideal gas laws

$p = \frac{F}{A}$

$pV = nRT = Nk_BT$

$E_K = \frac{3}{2}k_BT$

Oscillations and waves

$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricity

$I = \frac{Q}{t}$

$F = k \cdot \frac{q_1 q_2}{r_2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism

$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion

$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation

$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics

$E = hf$

$\lambda = \frac{hc}{E}$

Optics

$n_1\sin\alpha_1 = n_2\sin\alpha_2$

$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$

Question 1: Gravitation(6+6+8=20)

The potential energy of a mass m in a gravitational field like that of the earth is:

$$V(h) = -G\frac{Mm}{R+h}.$$

$M = 5{,}97 \times 10^{24}$ kg is the mass of the earth, $G = 6{,}6 \times 10^{-11}$ $\mathrm{Nm}^2/\mathrm{kg}^2$ is the gravitational constant, $R = 6{,}3 \times 10^{6}$ m is the radius of the earth, and h indicates the height of the mass m above the surface of the earth.

1) Close to the surface, we know that the potential energy of the mass m is also given by $V_{pot}(h) = mgh$. Calculate the constant g from the potential energy equation using the approximation $h \ll R$. (6P)

2) Suppose that a mass m on the surface of the earth is projected vertically with an initial speed of v_0. The minimum velocity allowing it to escape earth's gravitational field is called "escape velocity". Calculate this speed using energy conservation by assuming that the mass must be able to reach a point at an infinite distance from the earth. (6P)

3) To avoid the cost of multiple rockets, one could consider building a long ladder to space. If such a vertical ladder was available, what would its minimum height H be, so that an object that has reached the top of the ladder will not fall back to the ground after detaching from the ladder? (8P)

Question 2: proton in field E and B (6+6+3+3=18)

A proton is released at a point A, at $t = 0$ with no initial speed in a uniform and horizontal electric field of magnitude $E = 500\frac{V}{m}$. Friction and the weight of the proton can be neglected.

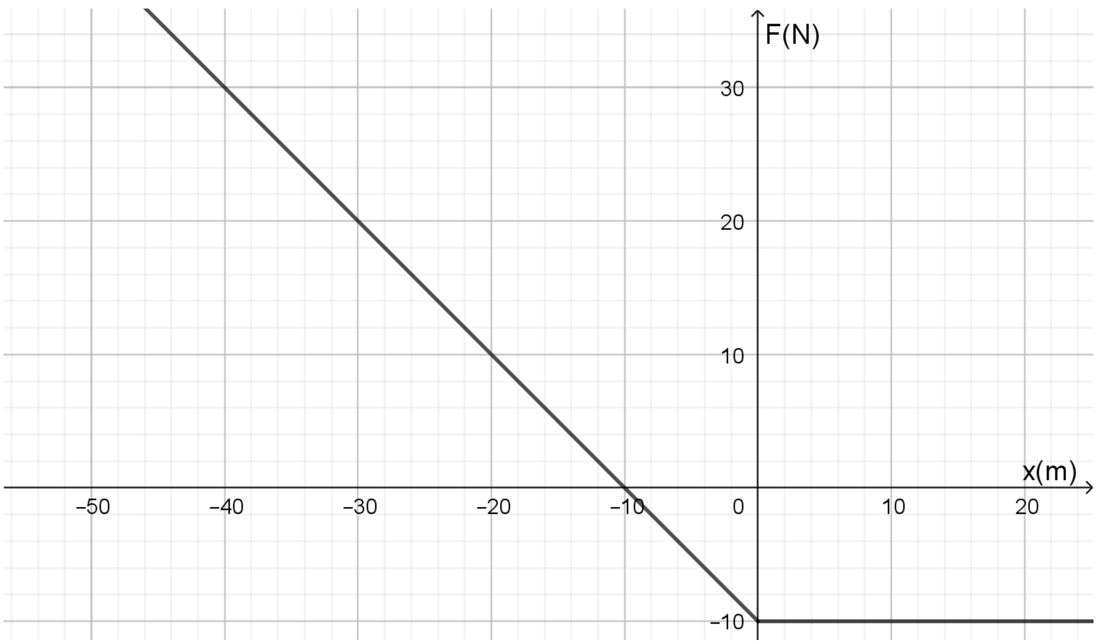
1) At what moment t_1, will the proton have acquired the speed of $10^5\ m/s$? How far did it travel? What is the potential difference U_{AB} between starting point A and arrival point B? What is its potential electric energy in B knowing that the corresponding energy at A was zero? What is its kinetic energy in B, expressed in eV?
2) At the time t_1, the proton leaves the electric field and enters a uniform and vertical magnetic field of magnitude $B = 0{,}1\ T$. What is the acceleration of the proton? How does its velocity evolve? What plane is its trajectory in?

We repeat the experiment, but this time, the proton is released in A, at $t = 0$, without initial speed in a region where the two previous fields are present simultaneously.

3) What will be the trajectory of the proton (line, parabola, circle or other)? Justify the answer.
4) Make a sketch of the trajectory, with the proton at a moment $t > 0$ and draw the forces on the proton as well as the speed and the acceleration.
5) Calculate the magnitude of the two forces when the speed of the proton is $10^5\ m/s$.

Question 3: One-dimensional motion (3+3+3+5+4+2=20)

A 1 kg point mass starts from rest at a point $x = 10\ m$ under the influence of a force $\vec{F}$. Friction is negligible. The force F depends on the coordinate x as shown below.



1) Calculate the speed of the mass at $x = 0$.
2) Describe the motion of the mass between $x = 10m$ and $x = -30m$.
3) Calculate the maximum kinetic energy. (Rep.: 150 J)
4) Make a graph of the kinetic energy E_c as a function of x between $x = 10\ m$ and $x = -30\ m$ indicating important points and values on the graph.
5) Make a graph of the speed as a function of time.
6) If additionally a frictional force of $F_f = 1\ N$ acts when $x \geq 0$ what is the maximum speed reached?

Question 4: Crookes Radiometer (2+8+4+2+2+3=21)

The Crookes radiometer is a device for visualizing heat radiation. When the device is exposed to electromagnetic radiation, the wings rotate because of the interaction between the wings and the gas molecules (assumed to be monoatomic).

Ideal 1-dimensional gas:

To simplify the analysis, it is assumed that the atoms of a ideal gas can only move in x direction (left or right).

1. A gas atom collides with a very heavy surface wall S at rest and oriented perpendicular to the velocity of the atom. Assume that the collision is elastic. Express the momentum transferred to the wall as a function of the mass m of the atom and the initial velocity v. (2P)
2. It is assumed that each atom of the ideal gas moves with the same v speed. Show that the kinetic energy of an atom must be $E_{cin} = \frac{1}{2} k_B T$ for the law of ideal gases to be valid ($PV = N k_B T$). Explain your reasoning based on your result from the previous question. (8P)

A Crookes radiometer wing is modeled by a thin plate of mass $M$and surface S, one side of which is in thermal equilibrium with the surrounding gas $T_1 = T_g$ while the other side is at a higher temperature $T_2 = 16\, T_1$(this is an exaggerated temperature difference to clarify the operating principle). The thermal conduction between the two parts is neglected.

On the heated side of the wing, a gas atom leaves with a kinetic energy corresponding to the temperature of that surface.

3. Express the resulting pressure difference between the sides of the stationary wing as a function of the initial gas pressure P (4P)
4. Discuss how this pressure difference changes as the wing begins to move. (2P)
5. What factors limit the speed of the radiometer's rotation? (Indication: The optimal pressure for the operation of the radiometer is in the order of 1 Pa) (2P)
6. What will be the maximum speed (neglecting friction) of the wings in an argon atmosphere at room temperature? $\left(M_{Ar} = 39{,}948 \frac{g}{mol}, T = 300K\right)$ (3P)