PHYSIKSOLYMPIAD
LËTZEBUERG 2019

Final (Theory) (EN)

30.03.2019

Formulae

Kinematics (UAM)

$x = \frac{1}{2}at^2 + v_0t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces

$F = ma$

$F_f \leq \mu N$

Work, Energy, Power

$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts

$Q = mc\Delta\theta$

$Q = mL$

Ideal gas laws

$p = \frac{F}{A}$

$pV = nRT = Nk_BT$

$E_K = \frac{3}{2}k_BT$

Oscillations and waves

$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricity

$I = \frac{Q}{t}$

$F = k \cdot \frac{q_1q_2}{r^2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism

$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion

$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation

$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics

$E = hf$

$\lambda = \frac{hc}{E}$

Optics

$n_1\sin\alpha_1 = n_2\sin\alpha_2$

$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$

Question 1 : Field ionisation(3+3+4+8+3=21P)

In this question, only classical mechanics will be applied. The proton and the electron are treated as point charged particles.

It is assumed that the electron follows a uniform circular motion around the proton (radiation losses are neglected). Knowing that the Bohr radius of the hydrogen atom is $a_0 = 0{,}0529\, nm$.

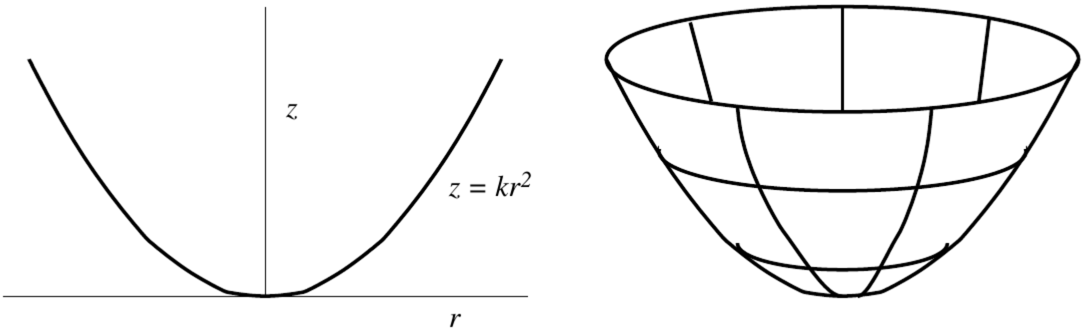
1. Calculate the linear speed of the electron.
2. The hydrogen atom is moving in an uniform electric field $\vec{E}$, perpendicular to the plane of the trajectory of the electron. It is assumed that the electron continues to move in a circular motion of radius a_0 perpendicular to the electric field, but in a plane shifted by a distance d with respect to the proton. Draw a detailed diagram of the situation. Label the forces acting on the electron.
3. Express the component parallel to the external field of the attraction force on the electron as a function of a_0, e, ϵ_0 and the distance d between the proton and the plane of the electron trajectory.
4. Determine the distance d_m for which the component of the force parallel to the field is a maximum.
5. Calculate the theoretical maximum electric field required to ionize the hydrogen atom and compare it to the field required to create an electric discharge in the air $E_{discharge} \approx 3\,\frac{kV}{mm}$.

A classical model does not make it possible to calculate the ionization by field effect. A complete derivation requires quantified energy levels as well as the quantum tunneling effect.

Question 2: Oscillations in a bowl(3+4+3+4+6+2=22P)

Source: United States Physics Team 2011 Semi-Finals.

A particle is constrained to move on the inner surface of a frictionless parabolic bowl whose cross-section has equation $z = kr^2$. The particle begins at a height z_0 above the bottom of the bowl with a horizontal velocity v_0 along the surface of the bowl. The acceleration due to gravity is g



Figure

Give an algebraic answer, unless otherwise indicated.

1. Consider first the case where the height of the body does not change over time. What forces are exerted on the body? Draw a diagram of all the forces acting on the body.

2. For a given initial velocity v_0 the height z of the body does not change (The trajectory is thus circular). Express v_0 according to g, z_0 and k.

3. Describe and explain the evolution of the system if the body has an initial velocity $v_0' > v_0$
4. Assuming that the angular momentum is conserved, determine the maximum height reached by the body. The conservation of the angular momentum means that the further the body moves away from the axis of rotation, the more it must slow down: $v_0 \cdot r_0 = v \cdot r$.

Consider now the case of a body with zero initial velocity $v_0 = 0$ and an initial height z_0.

5. Find the period of oscillation of the body around the equilibrium position at the bottom of the bowl, assuming that z_0. is small. (Tip: Depending on your approach, it may be useful to use the small angle approximation $\sin x \approx \tan x$ or to consider the different components of the velocity vector.)
6. For larger z_0 heights, will the period be equal to, greater than, or less than that obtained in the previous question? (Do not make complicated calculations, but you have to explain the answer qualitatively)

Question 3: Light and waves(2+1+2+1+1+1+1+2+2+2=15P)

A source emits waves which travel at speed c. Wave fronts leave the source with a frequency f.

1. A receiver is moving towards the source with a speed v. Determine the frequency f' at which the receiver encounters wave fronts.

We will assume that this equation for the Doppler effect holds for light waves with slow moving observers. We know that for a photon:

$$E = hf \text{ and } p = \frac{hf}{c}$$

So the energy and momentum of a photon as seen by moving observer is changed because of the relative motion of the source and observer. This fact is the basis for the technique of laser cooling, where a low pressure gas is bombarded with a monochromatic laser whose frequency is f_0 in order to reduce the temperature of the gas.

The gas atoms have internal quantified energy levels which allow for a transition ΔE which is slightly higher than the photon energy.

2. At which speed v do the gas atoms have to move towards the light source in order to fulfill the resonance condition $\Delta E = hf'$, where f' is the photon frequency as seen by the atom.
3. A gas atoms of mass m absorbs a photon absorbs a photon at the resonance condition. Calculate the new speed v' of the atom in the laboratory frame.
4. The atom may reemit a photon in the direction of the light source, while moving at speed v'. Calculate the photon frequency in the lab frame.
5. What is the final speed v_f of the atom in this case?
6. It is also possible for the excited atom to emit a photon in the opposite direction of motion. Calculate the emitted photon frequency and final speed of the atom in the lab frame. (Use v' as the initial velocity of the atom).
7. What is the overall result from an absorption/emission process?
8. How could you build a setup that cools a certain volume of gas using lasers?