PHYSIKSOLYMPIAD LËTZEBUERG 2019

Semi-final (EN)

26.02.2019

Instructions

- The exam includes 5 questions. To get full marks you have to answer all questions.
- Clearly indicate which sub-question you are answering and write you name on each sheet.
- Answer the questions by explaining your steps clearly.

Formulae

Kinematics (UAM)

$x = \frac{1}{2}at^2 + v_0 t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces

$F = ma$

$F_f \leq \mu N$

Work, Energy, Power

$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts

$Q = mc\Delta\theta$

$Q = mL$

Ideal gas laws

$p = \frac{F}{A}$

$pV = nRT = Nk_BT$

$E_K = \frac{3}{2}k_BT$

Oscillations and waves

$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricity

$I = \frac{Q}{t}$

$F = k \cdot \frac{q_1 q_2}{r_2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism

$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion

$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation

$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics

$E = hf$

$\lambda = \frac{hc}{E}$

Optics

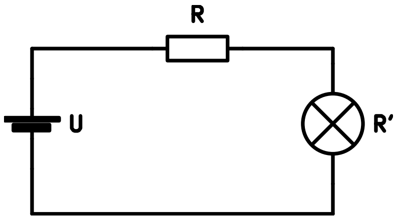
$n_1 \sin\alpha_1 = n_2 \sin\alpha_2$

$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$

Question 1: Electro-Magnetism 21P

Electricity suppliers, such as Creos, try to minimize the loss of electrical power in their grid. This is the reason why they use high voltage lines to transport energy over great distances.

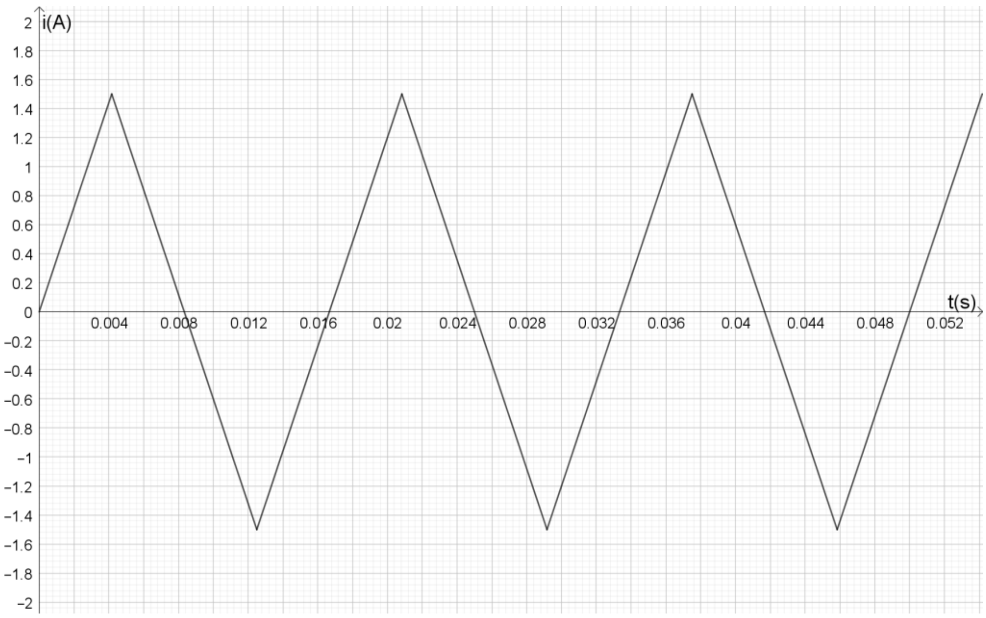
1. We model the high voltage line by a resistance R, the power plant by a DC voltage source U and the processing station by a lamp of resistance R'. These components are connected in series. Derive a relationship between R And R' in order to guarantee transport losses of less than 1% of the power supplied. Explain your reasoning. (4P)



2. Suppose the lamp has a power P. It is assumed that the potential difference across the lamp terminals is approximately equal to that of the power station $U' \approx U$. Find a condition or U in relation to R And P in order to satisfy the same condition as before for the transport losses. (4P)
3. What are the factors that companies can vary in order to minimize losses? (2P)

In order to refine the model, it must be taken into account that electric grid is based on alternating current (AC). In this case, components such as capacitors and coils behave differently, which has an effect on transport losses.

Imagine an electric motor running without mechanical resistance. This can be modeled by a coil with inductance $L = 10\ mH$ and negligible electrical resistance. The coil is connected to a triangular signal generator with a frequency of $f = 60\ Hz$. We measure the current i passing through the coil as a function of time.



4. Draw a circuit diagram with an arrow indicating the positive direction of flow of the current i and the voltage u across the coil. (2P)

5. Draw the voltage $u(t)$ as a function of time on the previous graph (with $i(t)$). Clearly indicate the scale you use. (4P)
6. On the graph above, label the intervals during which the inductor acts either as a receiver or as a generator. (1P)
7. What is the average power provided over one full period? (1P)
8. Explain the impact of the use of an electric motor for electricity suppliers from the point of view of the efficiency of the power grid. How could this impact be reduced? Explain your answer. (3P)

Answer Question 1

Question 2: Exoplanets 16P

Data:

1 AU (Astronomical Unit) = 150 million km

Value of the universal gravitational constant G = 6.67 · 10^{-11} N·m²·Kg⁻²

Sun Mass M_S = 1.99 · 10^{30} kg

Fomalhaut B is the first exoplanet detected under visible light by the Hubble Space Telescope. It is in orbit around the star Fomalhaut A, the brightest star in the constellation of Pisces. The figure below shows a copy of a photo with the star Fomalhaut A in the center and the exoplanet Fomalhaut B in the small square on the right. A part of its trajectory is shown in the square.

The scale of the photo is given at the bottom left. The scale of the box showing a part of the trajectory of the planet can be determined using the size of the small square. The enlarged part of the image shows the path the planet has followed between 2004 and 2006.

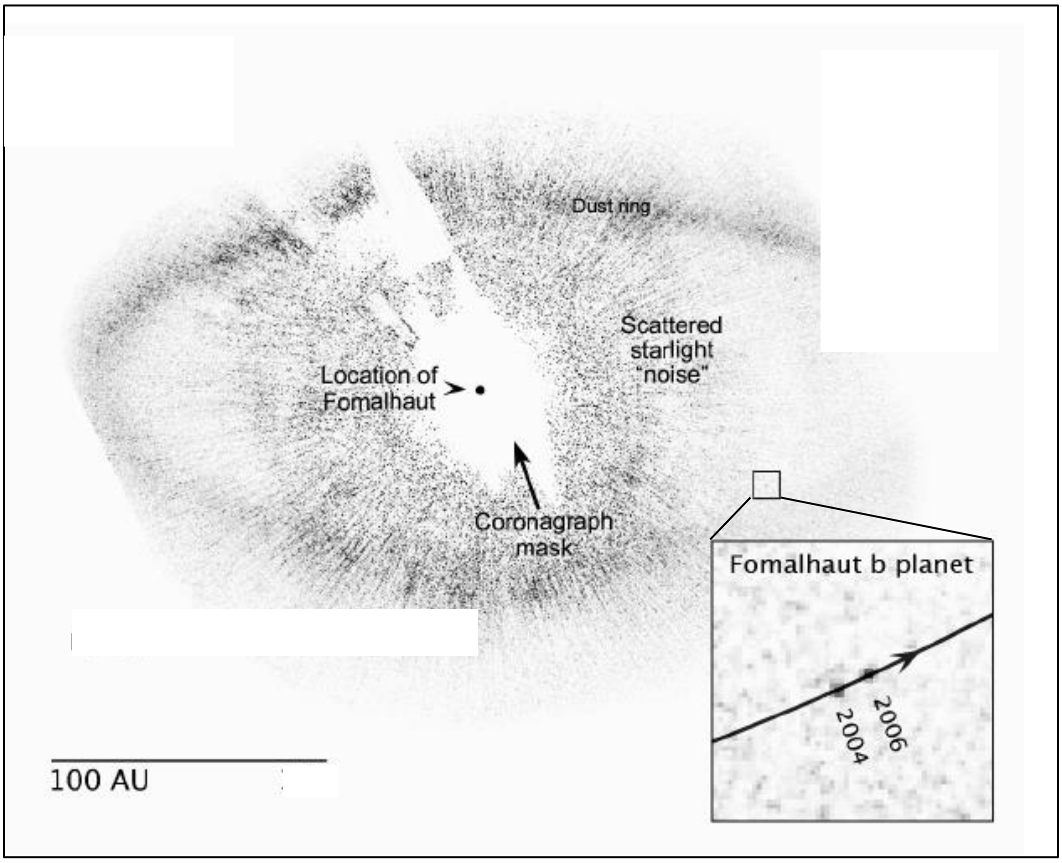


Figure 1 - The Hubble images were taken with the Space telescope Imaging Spectrograph in 2010 and 2012. Credit: NASA, ESA, and P. Kalas (University of California, Berkeley and SETI Institute)
Https://www.nasa.gov/mission_pages/hubble/science/rogue-fomalhaut.html

a) A planet revolves in a circular orbit of radius R and period T around a star. Derive an expression for the mass of the star as a function of R, T and G, the universal gravitational constant. You can assume that the mass of the planet is negligible in relation to the mass of the star. (4P)

b) In the calculation under a) it is assumed that the mass of the planet is negligible compared to that of the star. Explain why. (2P)
c) Use the image data to calculate the mass M_F of the star Fomalhaut as a multiple of the solar mass M_S. The orbit of the planet Fomalhaut B is considered circular. Reminder: The radius of the Earth's orbit around the sun is equal to 1AU. In your answer the number of significant figures should take into account the uncertainties of your measurements. (10P)

Answer Question 2

Question 3: Mechanics 20P

Collision between a tennis ball and a hard plate

A tennis ball is launched at a high speed against a horizontal plate that is non-deformable, and the ball's movement is recorded using a high-speed video camera.

6 interesting images are extracted from the video sequence. The series of 6 images, called 1, 2, 3, 4, 5 and 6, are shown on the left figure and correspond to the times noted from the top down t_1, t_2, t_3, t_4, t_5 and t_6. For simplicity, we may set $t_1 = 0$.

Images 1 and 2 correspond to the ball moving down; image 3 corresponds to the first contact with the plate, image 4 corresponds to the maximal deformation of the ball, on image 5 the ball just loses contact with the plate, image 6 shows to the ball moving up.

To answer the various questions, use the information provided by the 6 images, as well as the following numerical values: The mass of the ball is equal to M = 58.2 g, and its diameter d = 6.6 cm.

a) Knowing that the speed just before contact is 21.3 m/s determine the times t_2 and t_3. The speed is assumed to be constant over this time interval. (4P)

b) Knowing that the ball is compressed to a maximum after 2 ms of contact, determine the average acceleration of the ball during contact. Compare your results to the free fall acceleration on earth g. Determine the average force that the plate exerts on the ball, as well as the force which the ball exerts on the plate. (3P)

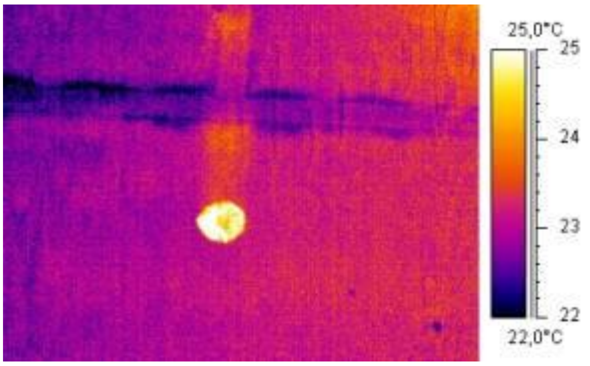
c) Determine the percentage of vertical deformation of the ball. (1P)

d) Knowing that the ball leaves the plate with a kinetic energy of 3.4 J, determine the corresponding velocity of the ball. (3P)

e) Determine the time t_6. (3P)

f) Determine the energy transformed into heat during the shock. By how many degrees Celsius could we warm up 1 cm^3 of water with this energy. The specific heat capacity of the water is $4180 \frac{J}{kg \cdot K}$? (5P)

g) The other picture is a long-exposure infrared photograph of the tennis court taken from the same perspective as the images of the video. The clear disk is the area of the ground heated by the impact of the ball. Give an interpretation for the vertical light red band. (1P) (Http://www.pro-physik.de/details/phiuznews/1305309/Von_Baellen_und_Schlaegern.html)



Answer Question 3

Question 4: Oscillations 24P

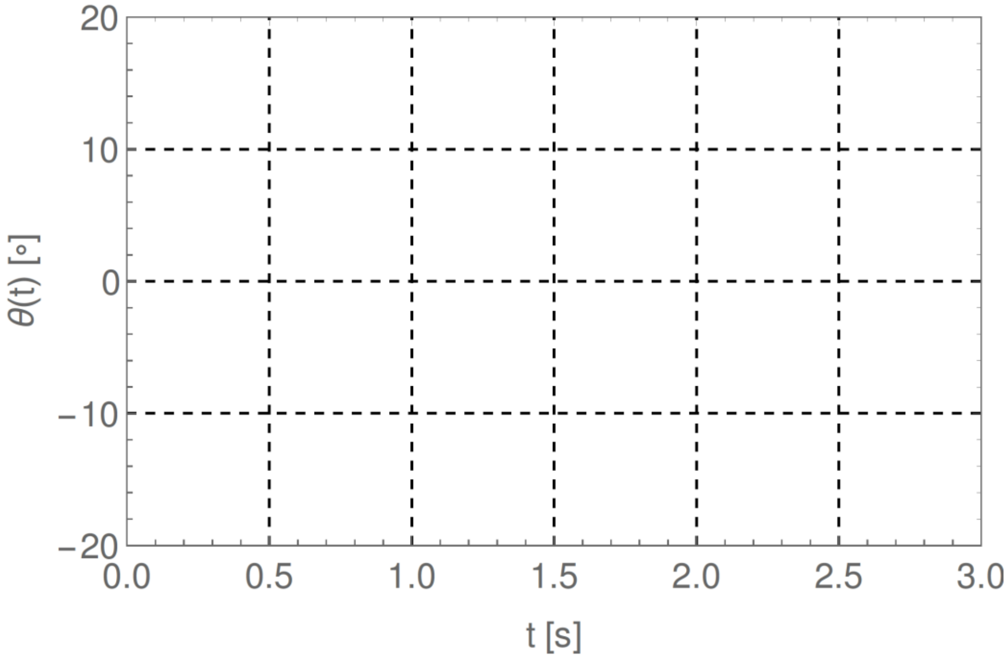
A mass $m = 0{,}1\ kg$ is attached to a string of length L and negligible mass. This system is a simple pendulum that performs oscillations around its equilibrium position.

1. On a diagram, draw the pendulum in a position other than its equilibrium position. Define a positive direction of oscillation and draw the forces acting on the mass, a coordinate system, as well as s measured from the equilibrium position.[1] (3P)
2. Establish the expressions of the tangential and normal acceleration. Express tangential acceleration using the circular arc length s measured from the equilibrium position. (3P)
3. Derive the differential equation for the angle $\theta(t)$ using the approximation of small angles ($\sin x \approx x$ if $x \ll 1$). What is the nature of the pendulum's movement? Reminder: The expression of the arc length is $s = L \cdot \theta$ where θ is the angle between the rope and the vertical. (2P)
4. Show that
$$\theta(t) = \theta_m \sin(\omega t + \theta_0)$$
is a solution of the differential equation found under 3 and give the condition for this to be true. (3P)
5. Show that the expression of the pendulum period is: (2P)
$$T = 2\pi\sqrt{\frac{L}{g}}$$

[1] Note that $s = 0$ at the equilibrium position. Watch out for the sign of s!

6. Exercise:

   a. Assume the equation of a simple pendulum is
   $$\theta(t) = 0.1\pi \sin\left(2\pi t + \frac{\pi}{2}\right).$$
   Sketch the graph of θ (in degrees) as a function of the time t (s) below. (2, 5p)



   b. Sketch, not to scale, the curve representing the speed $v(t)$ on the same chart. (1, 5p)

   c. Calculate the length L of the rope. (1, 5p)

   d. Calculate the speed of the pendulum at time $t = 1{,}3$ s. (1,5p)

   e. Calculate the tension in the rope at time $t = 1{,}3$ s. (3P)

   f. If we were to increase the length of the rope, while leaving the amplitude θ_m constant, how would the graph of $\theta(t)$ change? (1P)

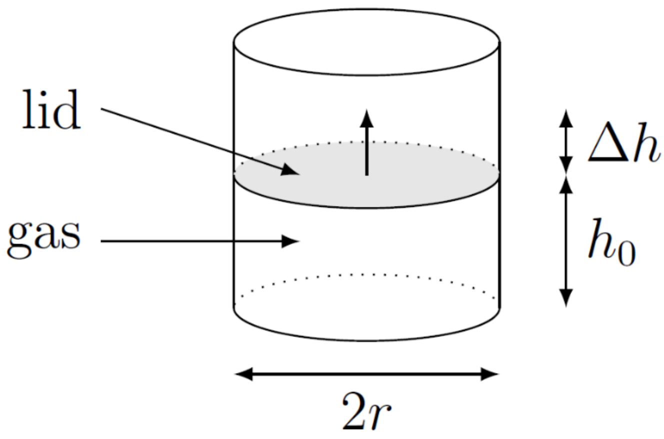
Answer Question 4

Name:________________________

Question 5: Thermodynamics

Heating a gas by laser irradiation

Consider a cylindrical container (radius $r = 10\ \mathrm{cm}$) whose lid (mass $m = 500\ \mathrm{g}$) is made of glass. It can move without friction along the vertical. The container contains an ideal gas of molar heat capacity $C_V = 20.8\ \mathrm{J/(mol \cdot K)}$. The light of a laser can pass through the lid and is then completely absorbed by the gas which heats up as a consequence.



The device is in an environment of temperature $T_{ext} = 20°C$ and the external pressure is $p_{ext} = 101{,}3\ kPa$. Before irradiation, the gas in the container is in equilibrium with this environment. The lid is $h_0 = 30\ \mathrm{cm}$ above the bottom of the container.

The laser is turned on for a duration $\Delta t = 10\ \mathrm{s}$. After this irradiation we find that the lid has risen by $\Delta h = 10\ \mathrm{cm}$.

a) What are the temperature T_0 and pressure p_0 of the gas before irradiation? (4P)

b) What are the temperature T_1 and pressure p_1 of gas after irradiation? (5P)

c) What work has the gas done to raise the cover? (4P)

d) How much energy was absorbed by the gas during this process? (5P)

Answer Question 5