PHYSIKSOLYMPIAD LËTZEBUERG 2023

Semi-final (EN)

02.02.2023

Instructions:

- Write your full name and school on each sheet.
- Clearly indicate which sub-/question you are answering.
- Explain your reasoning and indicate intermediary calculations.
- Number your pages.

Formulae

Kinematics (UAM)

$x = \frac{1}{2}at^2 + v_0t + x_0$

$v = at + v_0$

$v^2 - v_0^2 = 2a(x - x_0)$

Forces

$F = ma$

$F_f \leq \mu N$

Work, Energy, Power

$W = Fd\cos\theta$

$E_{cin} = \frac{1}{2}mv^2$

$E_{pes} = mgh$

$E_{el} = \frac{1}{2}kx^2$

$P = \frac{W}{t} = Fv$

Momentum

$p = mv$

$F = \frac{\Delta p}{\Delta t}$

Thermal concepts

$Q = mc\Delta\theta$

$Q = mL$

Ideal gas laws

$p = \frac{F}{A}$

$pV = nRT = Nk_BT$

$E_K = \frac{3}{2}k_BT$

Oscillations and waves

$T = \frac{1}{f}$

$c = f\lambda$

$T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

Electricity

$I = \frac{Q}{t}$

$F = k \cdot \frac{q_1q_2}{r_2}$

$V = \frac{W}{q}$

$E = \frac{F}{q}$

$V = RI$

$P = VI = RI^2 = \frac{V^2}{R}$

$R = R_1 + R_2 + \cdots + R_n$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$

$\rho = \frac{RA}{L}$

Electro-magnetism

$F = qvB\sin\theta$

$F = BIL\sin\theta$

Circular motion

$v = \omega r$

$a = \frac{v^2}{r}$

Gravitation

$F = G\frac{mM}{r^2}$

$g = \frac{F}{m}$

Quantum physics

$E = hf$

$\lambda = \frac{hc}{E}$

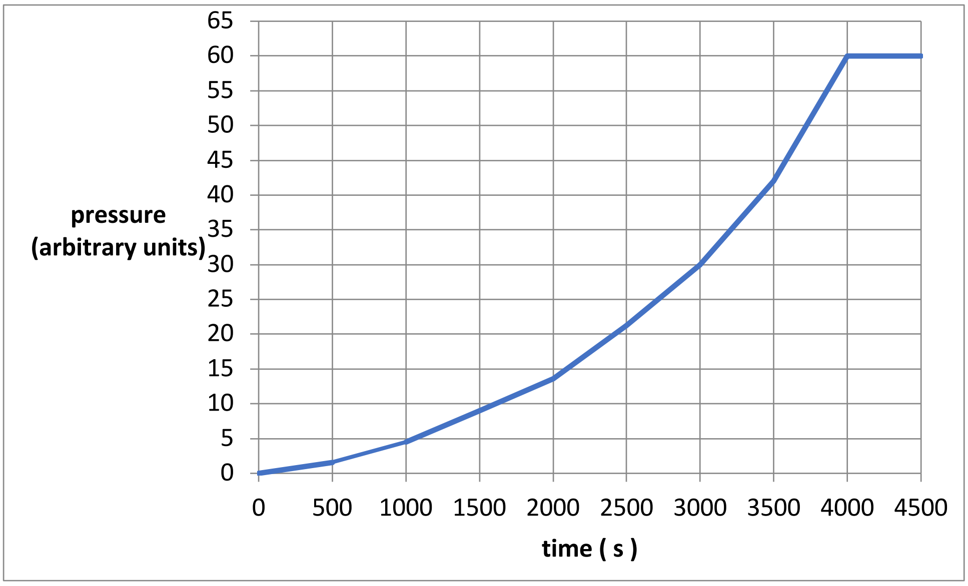
Optics

$n_1\sin\alpha_1 = n_2\sin\alpha_2$

$$\frac{1}{q} + \frac{1}{p} = \frac{1}{f}$$

Question 1: space probe (20 marks)

Upon entering the atmosphere of an unknown planet, a probe descends straight down to the surface. Along the way it recorded the atmospheric pressure as a function of time as shown in diagram below:



Unfortunately, the calibration of the pressure gauge has been lost and the units on the pressure axis are not known. Your mission, should you choose to accept it, is to compensate for this lack of calibration.

The atmosphere is mostly carbon dioxide with a molecular mass of 44 g/mol. The surface temperature measured by the probe is T_S = 400 K. The gravitational field g_S at the surface is 9.9 N/kg. The radius R of the planet is 5000 km.

1) Apply Newton's second law to a small layer of the atmosphere of vertical thickness Δy to show that the change in pressure Δp between top and bottom of the layer is given by

$$\Delta p = \rho g \Delta y,$$

where ρ is the density of the atmosphere and g is the local gravitational field. (4)

2) a) The formula above gives you the change in pressure with altitude. Locally the atmosphere can be treated as an ideal gas. Use the ideal gas law and the gradient (slope) of the pressure versus time graph at the surface to estimate the probe's speed v_S just before it strikes the surface. (5)

   b) Why is the calibration data not needed? (1)

3) Assuming the probe's speed is constant during its travel through the lower atmosphere, estimate the temperature of the atmosphere at a height of 15 km above the surface. (4)

4) It has been assumed that the speed is constant over the last 15 km before the probe hits the surface. Discuss how the temperature calculated in 3) would vary if

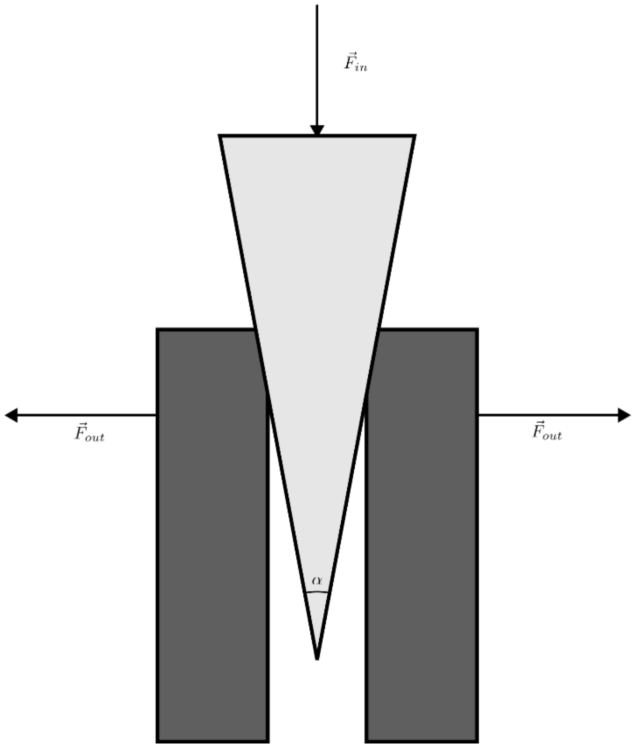
   a) the probe has already reached v_S before the 15 km altitude and (3)

   b) the probe has not yet reached v_S before the 15 km altitude. (3)

Question 2: forces and thermodynamics (20 marks)

1 - Stone cleaving

A block of marble of size $1\ \mathrm{m} \times 1\ \mathrm{m} \times 1\ \mathrm{m}$ must be cut into 2 pieces. To do this, holes with a diameter of 2 cm are drilled every 10 cm along a central line and wedges are inserted. The wedges are symmetrical and have an interior angle of $\alpha = 5°$. These wedges are hit with a hammer until the marble splits along the line of the holes.



Let's first study the effect of wedges in a static situation.

1) A force is applied $\vec{F}_{in}$ on the wedge as shown in the drawing. Friction between the block and the stone is neglected. Express the magnitude of the horizontal component F_{out} of the force exerted by the wedge on the marble. Explain your reasoning with a drawing. (**Hint:** without friction the contact force between the stone and the wedge must be normal to the contact surface) (3)

In practice, the situation is more dynamic. The stationary mass wedge is hit $m_1 = 0{,}2\ \mathrm{kg}$ with a mass hammer $m_2 = 2\ \mathrm{kg}$. The initial speed of the hammer is $v_2 = 10\ \mathrm{m/s}$.

2) Neglecting the interaction with the marble. What is the final velocity v' of the wedge assuming that the hammer remains stuck **to** the wedge (perfectly inelastic collision). (3)

3) The wedge and the moving hammer are then decelerated over a distance $d = 1\ \mathrm{mm}$ by the surrounding plate. Calculate the lateral force exerted by the block on the plate in this case. (3)

Marble cracks if pulled with a force of $10\ \mathrm{MN}$ per m^2 cross-sectional area.

4) In the situation described at the beginning of the exercise, imagine that each wedge is struck at the same time with a hammer at the same speed. Estimate the speed needed to split this piece of marble. (5)

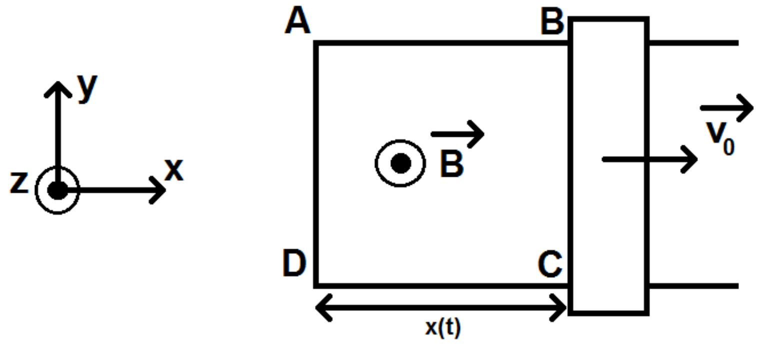
One could imagine achieving the same result by drilling holes through the block, adding dynamite which is detonated to split the stone. Let's suppose that the holes are perfectly sealed after the insertion of the dynamite.

5) What is the necessary pressure inside the holes to split the stone? (3)

6) Assuming that dynamite increases the number of gas molecules trapped in the holes by a factor of 100, calculate the temperature to which the gas mixture will need to be heated to split the stone. (3)

Question 3: electromagnetism (20 marks)

The following diagram is considered:

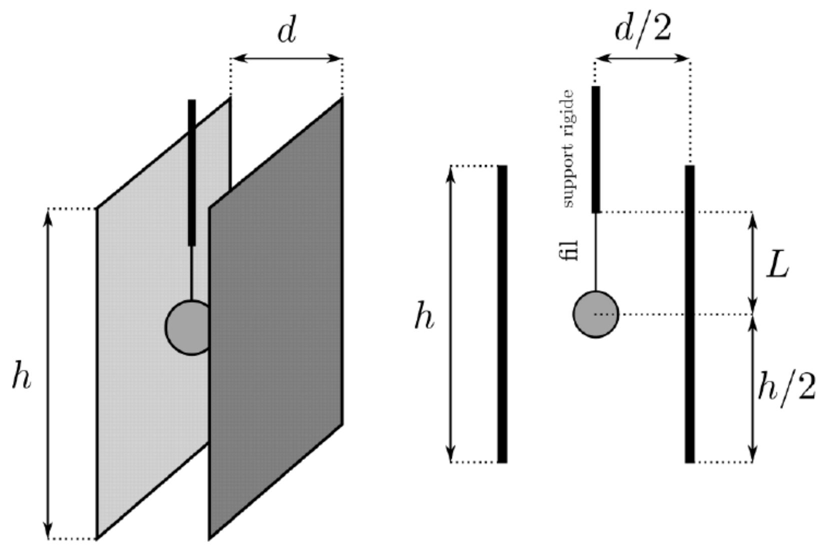


The circuit is closed by a fixed horizontal rail BADC, with no electrical resistance, and a moving conductor BC of length l, mass m and resistance R, which can roll/slide without friction along the axis (Ox). A magnetic field B directed along the axis (Oz) passes through the portion ABCD of the circuit. The conductor is thrown in the direction of increasing x and released at time t = 0 with a speed v_0.

1) Show that the speed v(t) of the moving conductor decreases. Explain your reasoning. (4)
2) Establish the expression for the acceleration of the moving conductor as a function of the characteristics of the conductor and the strength of the magnetic field B. (6)
3) Plot the approximate variation of v as a function of t. Explain briefly your graphical representation. Will the conductor reach complete rest? (4)
4) In what form is the initial kinetic energy dissipated over time? (1)
5) What should be the slope of the rail so that the conductor's speed remains constant over time? (5)

Question 4: electric pendulum (20 marks)

Consider a planar capacitor consisting of two vertical parallel plates, which cannot move. These plates are separated by a distance d, have height h and area $A \gg d^2$. In this problem, it will be assumed that the air resistance can be neglected.



View of the plane capacitor and initial position of the metal ball.

PART A: **Capacitor (3 marks)**

1) How does the capacitance C of the capacitor change if you double the distance d between the plates? (1)
2) If the air between the plates has a uniform electrical resistivity ρ, what will be the resistance between the plates? (1)
3) How does the energy stored in the capacitor change when the voltage U applied to the plates is doubled? (1)

PART B: **Constant voltage (17 marks)**

A metal ball of mass M and charge q is now suspended with a string connected to a rigid support. When the capacitor is not charged, the ball is in the centre of the capacitor (at a distance $d/2$ from each plate, and at a height $h/2$ above the bottom of the plates). On the other hand, if a voltage U is applied between the plates, the string will form an angle θ_0 with the vertical direction when the ball is in equilibrium.

1) Show that the expression for this angle in terms of the given quantities is (4)

$$\theta = arctan\left(\frac{qU_0}{Mgd}\right)$$

The metal ball is now lifted slightly, so that it is at an angle θ to the vertical direction, with θ only slightly larger than θ_0. The ball is then released.

2) Show that the period of oscillations of this harmonic motion as a function of the known quantities and fundamental constants is equal to (6)

$$T = 2\pi \sqrt{\frac{Lcos(\theta_0)}{g}}$$

Help: if the pendulum is subjected to an electric field $\vec{E}$ which is added to the gravity field $\vec{g}$, the oscillation can be considered to take place in an effective field $g_{new} = \frac{F_T}{M}$, where F_T is the tension in the string. $\vec{g}_{new}$ is collinear with $\vec{F}_T$.

3) What is the relationship between this period and the period the pendulum would have when there is no tension between the plates? (1)

When the ball is in equilibrium, the string is cut.

4) What is the motion of the ball? (1)
5) What is the maximum value of U_0 so that the ball does not touch a plate when it leaves the capacitor? Express your answer in terms of the given quantities and fundamental constants. (4)
6) How long would it take for the ball to leave the capacitor? (1)